Thematic section

ORVR

Ordered Random Variables and Reliability Theory

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SCHEDULE OF THE SECTION
Ordered Random Variables and Reliability Theory

- Monday – September 4th

  coffee break

  17:30–18:00 Mariusz Bieniek, *Comparison of L-statistics as quantile estimators*

  18:00–18:30 Andrzej Okolewski, *Sharp bounds on joint distribution functions of selected order statistics for k-independent observations*

  18:30–19:00 Tomasz Rychlik, *Bounds on the Expectations of Order Statistics for the Monotone Reversed Failure Rate Distributions*

- Tuesday – September 5th

  14:30–15:00 Jorge Navarro, *Preservation of aging classes in coherent systems*

  15:00–15:30 Magdalena Szymkowiak, *Necessary and sufficient conditions for preserving transform order properties of the component lifetimes by the system lifetime*

  15:30–16:00 Félix Belzunce, *Comparison of mean residual lives for dependent random variables*

  coffee break

  16:30–17:00 Maria Longobardi, *Relations between inactivity times of systems and lifetimes of their dual systems*

  17:00–17:30 Agnieszka Goroncy, *Number of components in the three-state* **k**-*out-of-n system*

  17:30–18:00 Krzysztof Jasiński, *On the number of failed components in a series-parallel system upon system failure*

Comparison of mean residual lives for dependent random variables

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Abstract

In this talk we consider two definitions of the mean residual life ordering of two random variables taking into account their mutual dependence. We prove several closure properties for the new definitions and some inferential issues are also considered for one of them.

Comparison of L-statistics as quantile estimators

Mariusz Bieniek

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Abstract

We consider the problem of quantile estimation by suitably chosen L-statistics. For a given sample size n we determine optimal L-statistics as estimators of the quantile of a given order $p \in (0,1)$. We use a new criterion of optimality, introduced in our paper [1], based on sharp bounds on the bias of the estimation. First we study the most popular cases of single order statistics and linear combinations of a pair of successive order statistics. Next, we study the mean square error of derived estimators and we compare them with well known L-statistics such as Kaigh-Lachenbrugh or Harell-Davis estimators.

[1] Bieniek M., Pańczyk L., *On the choice of the optimal single order statistic in quantile estimation*, Annals of the Institute of Statistical Mathematics 75 (2023), 303–333.

Number of components in the three-state k-out-of-n system

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joint work with Krzysztof Jasiński

Abstract

We consider a three-state **k**-out-of-n system with the independent and identically distributed component lifetimes. Such system and its components can function perfectly or totally fail, but can also enter a partial performance state, when they may not fail completely but their efficiency can be much reduced. Based on definitions introduced by Huang et al. (2000) and Tian et al. (2009) we focus on the random variables representing the numbers of components in such a system in respective states and compute the probability of the number of failed and partially operating components in the system in various settings.

[1] Huang J., Wu Y., Zuo M.J., *Generalized multi-state k-out-of-n: G systems*, IEEE Transactions on Reliability 49 (2000), 105–111.

[2] Tian Z., Yam R.C.M., Zuo M.J., *Multi-state k-out-of-n systems and their performance evaluation*, IIE Transactions 41 (2009), 32–44.

On the number of failed components in a series-parallel system upon system failure

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Abstract

We study reliability properties of a series-parallel system. We assume that the component lifetimes may be dependent and non-identically distributed (DNID) discrete random variables. We consider the number of failed components upon system failure. We derive the probability mass function and the expected value of this quantity, see [3] for more details. In addition, we find the conditional probabilities corresponding to this variate given some partial information about the system failure. The results correspond to those obtained by [1] and by [2].

[1] Davies K., Dembińska A., *On the number of failed components in a k-out-of-n system upon system failure when the lifetimes are discretely distributed*, Reliability Engineering and System Safety 188 (2019), 47–61.

[2] Dembińska A., Eryilmaz S., *Discrete time series-parallel system and its optimal configuration*, Reliability Engineering and System Safety 215 (2021).

[3] Jasiński K., *On the number of failed components in a series-parallel system upon system failure when the lifetimes are DNID discrete random variables*, Metrika, under review.

Relations between inactivity times of systems and lifetimes of their dual systems

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Abstract

The relation of the inactivity time of a system with structure function ϕ and the lifetime of a system with dual structure function ϕ^* is analyzed. The interesting case in which the lifetimes $T_1, \ldots, T_n$ of the components of the system S are related to the lifetimes $T_1^*, \ldots, T_n^*$ of the component of the dual system S^* by the relation $T_i^* = 1/T_i$, $i = 1, \ldots, n$, is studied in details. Some illustrative examples are given in terms of time-homogeneous load sharing models.

[1] Barlow R.E., Proschan F., *Statistical Theory of Reliability and Life Testing: Probability models*, Hold, Reinhart and Wiston, Inc. Silver Spring, MD, 1981.

[2] Buono F., Longobardi M., De Santis E., Spizzichino F., *Multivariate reversed hazard rates and inactivity times of systems*, Methodology and Computing in Applied Probability, 2021.

[3] Buono F., Longobardi M., Spizzichino F., *Inactivity times of systems and lifetimes of their dual systems*, in preparation (2023).

Preservation of aging classes in coherent systems

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Abstract

Different aging classes are used in probability theory to measure the effect of the time in the lifetime of a unit represented as a random variable. Some well known classes that represent the natural (positive) aging property are IFR (increasing failure rate), DMRL (decreasing mean residual life) and ILR (log-concave density). In all these classes the old units are worse than the young ones. When these units are placed in systems (series systems, parallel systems, k-out-of-n system) one would expect that the aging properties are transferred to the system. However, this is not always the case and we need some conditions to get these preservation properties. In this talk we will review some results in this direction and we will present some new ones.

[1] Alimohammadi M., Navarro J., *Resolving an old problem on the preservation of the IFR property under the formation of k-out-of-n systems with discrete distributions*, under review.

[2] Navarro J.: *Preservation of DMRL and IMRL aging classes under the formation of order statistics and coherent systems*, Statistics and Probability Letters 137 (2018), 264–268.

[3] Navarro J., *Introduction to System Reliability Theory*, Springer, 2022.

[4] Rychlik T., Szymkowiak M., *Signature conditions for distributional properties of system lifetimes if component lifetimes are i.i.d. exponential*, IEEE Transactions on Reliability 1–13 (2021).

Sharp bounds on joint distribution functions of selected order statistics for k-independent observations

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Abstract

We present pointwise sharp two-sided bounds on linear combinations of multivariate marginal distribution functions of order statistics based on k-independent identically distributed random variables. The model includes the model of arbitrarily dependent observations as particular case. Our results provide accurate expectation bounds for functions of selected order statistics from k-independent random variables with a finite set of values. Moreover, they lead to best-possible upper and lower bounds for the joint reliability function of any pair of semicoherent systems based on common k-independent components.

[1] Kemperman J.H.B., *Bounding moments of an order statistic when each $k-$tuple is independent*, in: Beneš V., Štěpán J., eds., Distributions with given marginals and moment problems, Dordrecht: Kluwer Academic Publishers, 1997, 291–304.

[2] Marichal J.-L., Mathonet P., Navarro J., Paroissin C., *Joint signature of two or more systems with applications to multistate systems made up of two-state components*, European Journal of Operational Research 263 (2017), 559–570.

[3] Rychlik T., *Bounds for expectations of L-estimates for dependent samples*, Statistics 24 (1992), 1–7.

Bounds on the Expectations of Order Statistics for the Monotone Reversed Failure Rate Distributions

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joint work with Agnieszka Goroncy
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Abstract

Danielak [1] and Goroncy and Rychlik [2] presented the sharp positive upper mean-variance bounds on the expectations of order statistics based on independent identically distributed random variables with the decreasing and increasing failure rates, respectively. In this paper we determine analogous evaluations in the dual cases when the parent distributions have monotone reversed failure rates.

[1] Danielak K., *Sharp upper mean-variance bounds for trimmed means from restricted families*, Statistics 27 (2003), 305–324.

[2] Goroncy A., Rychlik T., *Evaluations of expectations of order statistics and spacings based on IFR distributions*, Metrika 79 (2016), 635–657.

[3] Goroncy A., Rychlik T., *Bounds on the expectations of order statistics for the monotone reversed failure rate distributions*, submitted (2023).

Necessary and sufficient conditions for preserving transform order properties of the component lifetimes by the system lifetime

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Abstract

We consider coherent and mixed systems with identical components having a given structure function and a copula of dependence of component lifetimes which admits the Samaniego representation of the system lifetime distribution. We present natural necessary and sufficient conditions for preserving distributional properties of the component lifetimes by the system lifetime which are expressed by the relation of the respective distribution with some fixed life distribution in the star and superadditive orders. In particular, we provide conditions under which the system lifetime distribution preserves the IFRA, DFRA, NBU and NWU properties of component lifetimes.

[1] Arnold B.C., Rychlik T., Szymkowiak M., *Preservation of distributional properties of component lifetimes by system lifetimes*, TEST 31 (2022), 901–930.

[2] Navarro J., Rychlik T., Spizzichino F., *Conditions on marginals and copula of component lifetimes for signature representation of system lifetime*, Fuzzy Sets and Systems 415 (2021), 99–117.

[3] Rychlik T., Szymkowiak M., *Preservation of transform order properties of component lifetimes by system lifetimes*, in preparation.

[4] Shaked M., Shanthikumar J.G., *Stochastic Orders*, Springer, New York, 2007.