Thematic section

OA
Operator Algebras

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SCHEDULE OF THE SECTION
Operator Algebra

- Monday – September 4th

  16:00–16:30 Adam Paszkiewicz, *Linear combinations of projections and perturbations of projections in von Neumann factors of type* II_1

  16:30–17:00 Rafał Wieczorek, *Equality of entropies for states*

  coffee break

  17:30–18:15 Piotr Koszmider, *On singular pure states*

  18:15–18:45 Żywilla Fechner, *Moment functions and derivations on hypergroups*

- Tuesday – September 5th

  14:30–15:15 Antonio M. Peralta, *Metric invariants for unital Banach and Jordan–Banach algebras*

  15:15–16:00 Jan Hamhalter, *Maps preserving products of commuting elements between important structures on operator algebras*

  coffee break

  16:30–17:15 Bartosz Kwaśniewski, *Topologically free actions and ideals in twisted Banach algebra crossed products*

  17:15–17:45 Aleksandra Świątczak, *Delta-Sincov mappings in Banach algebras*

- Thursday – September 7th

  14:00–14:45 Joan Claramunt Carós, *A graph-theoretic characterization of a class of dynamical systems and its (C*)-algebras*

  14:45–15:30 Adam Skalski, *Characterising residually finite dimensional C*-algebras in dynamical contexts*

  15:30–16:00 Tomasz Brzeziński, *The C*-algebras of quantum lens and weighted projective spaces*

  coffee break

  16:30–17:15 Mateusz Wasilewski, *Quantum Cayley graphs*

The C^*-algebras of quantum lens and weighted projective spaces

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based on a joint work with Wojciech Szymański (Odense)

Abstract

We identify the algebras of continuous functions on quantum weighted projective and lens spaces as graph C^*-algebras. This is then used to compute the K-theory of these spaces in cases that have not been computed previously.

[1] Brzeziński T., Szymański W., *The C^*-algebras of quantum lens and weighted projective spaces*, Journal of Noncommutative Geometry 12 (2018), 195–215.




A graph-theoretic characterization of a class of dynamical systems and its (C^*)-algebras

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joint work with Pere Ara (Universitat Autònoma de Barcelona)

Abstract

We present a graph-theoretic model for dynamical systems given by a surjective local homeomorphism on a totally disconnected compact metric space. This construction gives a bijective correspondence between such dynamical systems and a subclass of two-colored Bratteli separated graphs.

We use this construction in order to write any dynamical system of our interest as an inverse limit of a sequence of (what we call) *generalized finite shifts*. This enables us to compute the associated Steinberg algebra (resp. C^*-algebra) of the dynamical system as colimits of the graph algebras (resp. graph C^*-algebras) associated with the different levels of the corresponding separated graph.

In subsequent work we plan to apply this theory to relate the type semigroup of the dynamical system with the graph monoid of the corresponding separated graph, and with the non-stable K-theory of the Steinberg algebra.

Moment functions and derivations on hypergroups

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joint work with Eszter Gselmann and László Székelyhidi

Abstract

The aim of this talk is to define higher order derivations and generalized moment generating functions in the hypergroup settings. The connection between these notions is given in the following theorem:

Let X be a commutative hypergroup and r a positive integer. The family $(D_\alpha)_{\alpha\in\mathbb{N}^r}$ of self-mappings on $\mathcal{M}_c(X)$ is a continuous higher order derivation of order r if and only if there exists a generalized moment function sequence $(\varphi_\alpha)_{\alpha\in\mathbb{N}^r}$ of rank r such that

$$\langle D_\alpha\mu, f\rangle = \int_X f\cdot\varphi_\alpha\, d\mu$$

holds for each μ in $\mathcal{M}_c(X)$, f in $\mathcal{C}(X)$ and α in $\mathbb{N}^r$.

The proof can be found in [2].

[1] Bloom W.R., Heyer H., *Harmonic analysis of probability measures on hypergroups*, de Gruyter Studies in Mathematics, vol. 20, Walter de Gruyter & Co., Berlin 1995.

[2] Fechner Ż, Gselmann E., Székelyhidi L., *Moment functions and exponential monomials on commutative hypergroups*, Aequationes Mathematicae 95 (2021), 1281–1290.

[3] Fechner Ż, Gselmann E., Székelyhidi L., *Moment Functions on Groups*, Results in Mathematics 76 (2021), no. 4.

Maps preserving products of commuting elements between important structures on operator algebras

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Abstract

Bijective bicontinuous maps preserving products of commuting elements (in both directions) that act between positive definite cones of von Neumann algebras or between unitary groups of von Neumann algebras are described in terms of the following parameters: (i) Jordan *-isomorphism between given algebras (ii) fixed central element (iii) appropriate form of central valued character. Especially, in the case of unitary groups of von Neumann factors, such maps are precisely either Jordan *-isomorphisms or Jordan *-isomorphisms composed with the inverse map $u \to u^{-1}$. These advances lead to new complete Jordan invariants of operator algebras. Besides, we present related general results on commutative parts of operator algebras and Banach algebras.

[1] Hamhalter J., *Piecewise *-homomorphisms and Jordan maps on C*-algebras and factor von Neumann algebras*, Journal of Mathematical Analysis and Applications 462 (2018), 1014–1031.

[2] Hamhalter J., *Maps preserving products of commuting elements in von Neumann algebras*, Journal of Mathematical Analysis and Applications 523 (2023).

[3] Molnár L., *General Mazur-Ulam type theorems and some applications*, in "Operator Semigroups Meet Complex Analysis, Harmonic Analysis and Mathematical Physics", Arendt W., Chill R., Tomilov Y. (Eds.), Operator Theory: Advances and Applications 250, 311–342, Birkhäuser, 2015.

On singular pure states

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Abstract

Let $\mathcal{B}(\ell_2)$ stand for the C*-algebra of all bounded linear operators on ℓ_2 and $\mathcal{D}_E(\ell_2)$ for the atomic masa of the operators which are diagonal with respect to an orthonormal basis E.

The relation between pure states of $\mathcal{B}(\ell_2)$ and the pure states of the algebras $\mathcal{D}_E(\ell_2)$ attracted interest at least since the seminal paper [3] of Kadison and Singer . In [2] J. Anderson conjectured that every pure state ϕ on $\mathcal{B}(\ell_2)$ is diagonalizable, that is, can be simply described by a choice of an orthonormal basis $E = (e_n)_{n\in\mathbb{N}}$ for ℓ_2 and a pure state u of $\mathcal{D}_E(\ell_2)$, i.e., for every $T \in \mathcal{B}(\ell_2)$ it satisfies

$$\phi(T) = \lim_{n\in u}\langle T(e_n), e_n\rangle,$$

where $\lim_{n\in u} c_n = c$ means that for every $\varepsilon > 0$ there is $A \in u$ such that for every $n \in A$ we have $|c_n - c| < \varepsilon$. Here we identify the pure states of $\mathcal{D}_E(\ell_2) \equiv \ell_\infty \equiv C(\beta\mathbb{N})$ with ultrafilters on $\mathbb{N}$. The nontrival case is of singular pure states, i.e., vanishing on the ideal of all compact operators on $\mathcal{B}(\ell_2)$ (not vector states).

In [1] Akemann and Weaver assuming the continuum hypothesis (CH) constructed a pure state on $\mathcal{B}(\ell_2)$ whose restriction to any masa is not a pure state on this masa (equivalently, the restriction is not multiplicative). This proved that CH implies that Anderson's conjecture is false, leaving open the question if Anderson's conjecture is consistent with the usual axioms ZFC as well. We present a new construction of a non-diagonalizable pure state which does not need any additional set-theoretic assumption and so disprove Anderson's conjecture in ZFC alone ([4]). By the solution of the Kadison-Singer problem this shows that the existence of pure states which are not multiplicative when restricted to any atomic masa also does not need any additional hypothesis.

[1] Akemann C., Weaver N., *$\mathcal{B}(\mathcal{H})$ has a pure state that is not multiplicative on any masa*, Proceedings of the National Academy of Sciences of the United States of America 105 (2008), 5313–5314.

[2] Anderson J., *A conjecture concerning the pure states of $B(H)$ and a related theorem* in Operator Theory: Advances and Applications, Birkhauser, Basel, Switzerland, 1981, vol. 2, 27–43.

[3] Kadison R.V., Singer I.M., *Extensions of pure states*, American Journal of Mathematics 81 (1959), 383–400.

[4] Koszmider P., *A non-diagonalizable pure state*, Proceedings of the National Academy of Sciences of the United States of America 117 (2020), no. 52, 33084–33089.

Topologically free actions and ideals in twisted Banach algebra crossed products

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joint work with Krzysztof Bardadyn [2]

Abstract

We generalize the well known C^*-algebraic result of Kawamura-Tomiyama [3] and Archbold-Spielberg [1] for crossed products of discrete transformation groups to the realm of Banach algebras and twisted actions. Namely, we show that topological freeneess is equivalent to the intersection property for all reduced twisted Banach algebra crossed products coming from subgroups, and in the untwisted case to a generalized intersection property for a full L^p-operator algebra crossed product for some (and hence any) $p \in [1, \infty]$. This gives efficient simplicity criteria for various Banach algebra crossed products. We also use it to identify the prime ideal space of some crossed products as the quasi-orbit space of the action.

It solves some of the open problems from [4].

[1] Archbold R.J., Spielberg J.S., *Topologically free actions and ideals in discrete C^*-dynamical systems*, Proceedings of the Edinburgh Mathematical Society 37 (1993), no. 2, 119–124.

[2] Bardadyn K., Kwaśniewski B.K., *Topologically free actions and ideals in twisted Banach algebra crossed products*, arXiv:2307.01685 (2023).

[3] Kawamura S., Tomiyama J., *Properties of topological dynamical systems and corresponding C^*-algebras*, Tokyo Journal of Mathematics 13 (1990), 251–257.

[4] Phillips N.C., *Crossed products of L^p operator algebras and the K-theory of Cuntz algebras on L^p spaces*, arXiv:1309.6406 (2013).

Linear combinations of projections and perturbations of projections in von Neumann factors of type II_1

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joint work with Stanisław Goldstein

Abstract

We present recent results of the following type: For any hermitian operator $a \in \mathcal{M}$, we have $a \in \mathrm{lin}(p_1, \ldots, p_n)$ for some projections $p_1, \ldots, p_n \in \mathcal{M}$; for some hermitian operator $a \in \mathcal{M}$, $a \notin \mathrm{lin}(p_1, \ldots, p_{n-1})$ for any projections $p_1, \ldots, p_{n-1} \in \mathcal{M}$. If proves that $n = 4$ for $\mathcal{M}$ being a factor of type I_n, $n > 76$; I_∞; II_1 or II_∞ but $n = 3$ for $\mathcal{M}$ of type III.

Some relations to the theory of conditional expectations in von Neumann algebras will be explained.

Some new look at old methods in perturbations theory of operator in Hilbert space will also be presented.

Characterising residually finite dimensional C^*-algebras in dynamical contexts

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Abstract

A C^*-algebra is said to be residually finite-dimensional (RFD) when it has 'sufficiently many' finite-dimensional representations. The RFD property is an important, and still somewhat mysterious notion, admitting several equivalent descriptions and having subtle connections to residual finiteness properties of groups. In this talk I will present certain characterisations of the RFD property for C^*-algebras of amenable étale groupoids and for C^*-algebraic crossed products by amenable actions of discrete groups, extending (and inspired by) earlier results of Bekka, Exel and Loring. I will also explain the role of the amenability assumption and describe several consequences of our main theorems. Finally I will discuss some examples, notably these related to semidirect products of groups.

[1] Shulman T., Skalski A., *RFD property for groupoid C^*-algebras of amenable groupoids and for crossed products by amenable actions*, arXiv:2305.12122 (2023).

Delta-Sincov mappings in Banach algebras

Aleksandra Świątczak

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Abstract

We study solutions and approximate solutions of the multiplicative Sincov equation

$$T(f,h) = T(f,g)T(g,h)$$

for mapping T taking values in a commutative Banach algebra.

Quantum Cayley graphs

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Abstract

I will propose a definition of a quantum Cayley graph of a discrete quantum group. The first step will be to extend the framework of quantum graphs to a very restricted infinite dimensional setting, namely the infinite direct sums of matrix algebras. For discrete quantum groups the natural candidates for quantum adjacency matrices will be convolution operators against a projection. If this projection satisfies a symmetry condition and is generating, then it can be used to define a quantum Cayley graph. These quantum graphs to a large extent do not depend on the generating projection, just like for classical groups. In the last part of the talk I will present some examples.

[1] Wasilewski M., *On quantum Cayley graphs*, arXiv:2306.15315 (2023).

Equality of entropies for states

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Abstract

In this talk we will deal with the Segal and Renyi entropy of a state on a semifinite or finite von Neumann algebra. We investigate the situation when two normalised states h_ρ and h_φ have equal Rényi's entropies $S_\alpha(\rho) = S_\alpha(\varphi)$ for α in some neighbourhood of 1, and when they have equal Segal's entropies $H(h_\rho + s\mathbb{1}) = H(h_\varphi + s\mathbb{1})$ or Rényi's entropies $S_\alpha(h_\rho + s\mathbb{1}) = S_\alpha(h_\varphi + s\mathbb{1})$ for some fixed α and a family of perturbed states with s in some interval $(c, d) \subset (0, +\infty)$. We will characterize these conditions.