Thematic section

ME
Mathematical Economics

ORGANIZERS:

Carlos Herves Beloso (Universidade de Vigo)

Marta Kornafel (Uniwersytet Ekonomiczny, Kraków)

Emma Moreno Garcia (Universidad de Salamanca)

Agnieszka Wiszniewska-Matyszkiel (Uniwersytet Warszawski)

SCHEDULE OF THE SECTION
Mathematical Economics

- Tuesday – September 5th

  14:30–15:00 Anna Jaśkiewicz, *Time-consistency in the mean-variance problem: a new perspective*

  15:00–15:30 Dariusz Zawisza, *Optimal portfolio selection on the bond market*

  15:30–16:00 Juan Pablo Rincon Zapatero, *TBA*

  coffee break

  16:30–17:00 Honorata Sosnowska, *Three voting methods diminishing manipulation tendency. Comparison of properties*

  17:00–17:30 Emma Moreno-García, *Economies with rights: efficiency and inequality*

  17:30–18:00 Carlos Hervés-Beloso, *Ordinar and cardinal preferences. Is there a single, canonical utility representation of cardinal preferences?*

  18:00–18:30 Lesław Gajek, *Ruin probabilities in regime - switching models with imprecise information about the switch*

- Thursday – September 7th

  14:30–15:00 Jakub Bielawski, *Chaos in learning dynamics*

  15:00–15:30 Dominika Machowska, *Closed-loop Nash equilibrium for a partial differential game with application to competitive personalized advertising*

  15:30–16:00 Agnieszka Wiszniewska-Matyszkiel, *A mathematical model of "the tragedy of the commons" with relation to counteracting pandemic*

  coffee break

  16:30–17:00 Łukasz Woźny, *A global version of Tarski-Kantorovitch theorem for correspondences*

  17:00–17:30 Agnieszka Lipieta, *Optimal demand-driven eco-mechanisms leading to equilibrium in competitive economy*

  17:30–18:00 Marta Kornafel, *An economic growth model with ecological investments*

Chaos in learning dynamics

Jakub Bielawski

Krakow University of Economics
email: bielawsj@uek.krakow.pl

Abstract

We study behavior of agents in a congestion game when the agents make choices of strategies according to a learning algorithm. We consider machine learning algorithms (Multiplicative Weights Update, Follow the Regularized Leader) and a behavioral algorithm (Experience Weighted Attractions). We show that when the size of the population of agents is small all trajectories of the system converge to an equilibrium (Nash Equilibrium or Quantal Response Equilibrium). However, when the size of the population of agents increases the equilibrium loses stability and we observe periodic/chaotic behavior of the agents. Moreover, the size of the chaotic regime is smaller in the behavioral algorithm in comparison to machine learning algorithms.

Ruin probabilities in regime – switching models with imprecise information about the switch

Lesław Gajek

Lodz University of Technology
Institute of Mathematics
email: leslaw.gajek@p.lodz.pl

Abstract

Regime-switching Markovian models can describe natural disasters such as floods more accurately then the corresponding models without a switch. However, when using switching models, we must first provide them with a transition matrix that can be estimated from the data. In this lecture we will investigate how much the inaccuracy in determining the transition matrix of the regime-switching Sparre Andersen model can affect the infinite-horizon ruin probability of the insurer.

Time-consistency in the mean-variance problem: a new perspective

Anna Jaśkiewicz

Politechnika Wrocławska
Wydział Matematyki
email: anna.jaskiewicz@pwr.edu.pl

joint work with N. Bäuerle [1]

Abstract

We investigate discrete-time mean-variance portfolio selection problems viewed as a Markov decision process. We transform the problems into a new model with deterministic transition function for which the Bellman optimality equation holds. In this way, we can solve the problem recursively and obtain a time-consistent solution, that is an optimal solution that meets the Bellman optimality principle. We apply our technique for solving explicitly a more general framework.

[1] Bäuerle N., Jaśkiewicz A., *Time-consistency in the mean-variance problem: a new perspective*, submitted (2023).

An economic growth model with ecological investments

Marta Kornafel

Krakow University of Economics
Department of Mathematics
email: marta.kornafel@uek.krakow.pl

Abstract

We will present an economic growth model, which takes into account the investments in creating an infrastructure for production and distribution of renewable energy. In the economy the energy is supplied from both nonrenewable and renewable resources. The dynamics of the model in the context of stability of equilibria will be analysed and economic conclusions drawn.

Optimal demand-driven eco-mechanisms leading to equilibrium in competitive economy

Agnieszka Lipieta

Krakow University of Economics
Department of Mathematics
email: alipieta@uek.krakow.pl

Abstract

We examine new mechanisms that introduce environmentally friendly eco-changes involving the elimination of noxious commodities and take into account the structure of demand without a detrimental effect to agents' position. In the era of the fourth industrial revolution, these mechanisms allow eliminating unnecessary services or goods that are being replaced by modern technologies. We define optimal mechanisms under the criterion of distance minimization, when a small number of detrimental commodities is excluded from production processes as well as when producers are change-averse. The results have the form of theorems with rigorous proofs.

[1] Denkowska A., Lipieta A., *Optimal demand-driven eco-mechanisms leading to equilibrium in competitive economy*, Central European Journal of Economic Modelling and Econometrics 14 (2022), 225–262.

[2] Lipieta A., Malawski A. *Price versus Quality Competition: In Search for Schumpeterian Evolution Mechanisms*, Journal of Evolutionary Economics 26 (2016), 1137–1171.

Closed-loop Nash equilibrium for a partial differential game with application to competitive personalized advertising

Dominika Machowska

Uniwersytet Warszawski
email: machowska@mimuw.edu.pl

Abstract

This paper is devoted to an N-person partial differential game whose dynamics of the state variable is described by a hyperbolic differential equation with certain boundary and initial conditions while the objective of each player is given by a finite horizon accumulated payoff functional with discounting. We extend the concept of a closed-loop Nash equilibrium for a partial differential game with the dynamics of the states described by a hyperbolic differential equation (a transport equation). We propose the definition of a dual closed-loop Nash equilibrium for which we give sufficient conditions. Moreover, we present the relationship between the Nash equilibria with the dual closed-loop and the classical closed-loop information structure. We apply the new results to the goodwill dynamics model in which the goodwill is influenced by personalized advertising and consumers' recommendations for which we construct a dual closed-loop Nash equilibrium and we examine its economic properties.

Economies with rights: efficiency and inequality

Emma Moreno-García

Universidad de Salamanca
email: emmam@usal.es

joint work with Carlos Hervés-Beloso

Universidad de Vigo. ECOBAS
cherves@uvigo.es
and Francisco Martínez Concha
Universidad de Chile
Institute of Complex Engineering Systems
fmartine@ing.uchile.cl

Abstract

Very large inequality levels originate negative externalities. This paper introduces, within a general equilibrium framework, tradable consumption rights to obtain efficient outcomes, reduce inequality, and improve social welfare. This mechanism would be easily implementable with the necessary support of the law.

Three voting methods diminishing manipulation tendency. Comparison of properties

Honorata Sosnowska

SGH Warsaw School of Economics
Institute of Mathematical Economics
email: honorata@sgh.waw.pl

Abstract

The small voting bodies, such as juries of classical music or sports competitions, are considered. In such bodies, very often, various types of rankings are used. Jury members may try to manipulate voting by giving high scores to their favorites and low scores to their opponents. Gibbard-Satherwaitte's, [1, 3], theorem shows that there is no voting method that preserves against manipulation, different from dictatorship. We can only find some methods which may diminish manipulation tendency. The most popular such methods are based on two methods, the Olympic Mean and winsorizing. Members of a jury give scores to the contestants.

The most known score system is the Borda Count. There are n contestants. Jurors give n points to the best, $n-1$ points to the second best, and so on till 1 point to the worst. For every contestant, the mean of jurors' scores is computed. The contestant with the highest mean wins. The Borda Count is very sensitive to manipulation.

The Olympic Mean is a kind of trimmed mean. One lowest score and one highest score are removed, and the mean of remained scores is computed. The contestant with the highest such mean wins. This method and its generalizations were used during the International Henryk Wieniawski Violin Competitions, the main violin competition in Poland.

Winsorizing is a method where reduction to a deviation from the mean is applied. For every contestant, the mean of jurors' scores M s is computed. There is established a deviation from the mean, a. The deviation is common for all contestants. If a score is lower than $M-a$, it is increased to $M-a$. If a sore is higher than $M+a$, it is reduced to $M+a$. The other scores remained unchanged. The mean of such improved scores is computed, and the contestant with the highest mean wins. This method and its generalizations were used during the International Fryderyk Chopin Piano Competitions, one of the main piano competitions in the world.

The third method is a new method introduced by Kontek and Sosnowska [2]. For every contestant, the mean of scores is computed. Then,

for every juror, the distance between the vector of the juror's scores and the vector of means is computed. 20% of jurors with the highest such distance are removed. The means of scores of the rest jurors are computed, and the contestant with the highest mean wins. This method was used in the International Chopin Competition for Amateurs.

The Borda Count is characterized by a set of conditions that it fulfils, and any method fulfilling these conditions is the Borda Count [4]. The same property has any scoring method [5]. It is a question whether the Olympic Mean, winsorizing, and anti-manipulation methods have such characterization. The question is open, but the properties of the method are studied. In this paper, we consider the following properties: consistency, vulnerability to the No-Show paradox, vulnerability to the Condorcet winner paradox, vulnerability to the Condorcet loser paradox, monotonicity, homogeneity, subset choice condition, and vulnerability to reversal bias. The methods are compared respectively, fulfilling these properties.

[1] Gibbard A., *A Pareto-consistent libertarian claim*, Journal of Economic Theory 7 (1974), 388–410.

[2] Kontek K., Sosnowska H., *Specific tastes or cliques of jurors. How to reduce the level of manipulation in group decisions?*, Group Decision and Negotiation 19 (2020), 1057–1084.

[3] Satterthwaite M.A., *Strategy-proofness and Arrow's condition: existence and correspondence for voting procedures and social welfare functions*, Journal of Economic Theory 10 (1975), 187–221.

[4] Young H.P., *Axiomatization of Borda's rule*, Journal of Economic Theory 9 (1974), 43–52.

[5] Young H.P., *Social Choice Scoring Functions*, SIAM Journal of Applied Mathematics 28 (1975), no. 4, 824–838.

A mathematical model of "the tragedy of the commons" with relation to counteracting pandemic

Agnieszka Wiszniewska-Matyszkiel

University of Warsaw
Institute of Applied Mathematics and Mechanics
email: agnese@mimuw.edu.pl

joint work with Rajani Singh
Copenhagen Business School, Denmark Department of Digitalization

Abstract

We model "the tragedy of the commons" when a scarce resource becomes a strategic input in production of a countermeasure to a sudden threat to the society's safety. The model uses the terminology of squalene market in the context of COVID-19 vaccines with adjuvants based on squalene obtained from endangered sharks and it is a dynamic game taking into account the privileged position of a relatively small number of vaccine producers.

From the economic point of view, the game describes a market consisting of pharmaceutical, cosmetic and fishing sector, in presence of a regulatory institution. From the mathematical point of view, we face coupled and hierarchical optimization problems of all the agents, who are elements of two continua and a discrete finite set. We consider Nash and Stackelberg equilibria in which COVID-19 vaccine producers do not take into account their influence on the population of sharks. Using topological and convex analysis tools, we prove the existence and uniqueness of equilibrium together with deriving the formula to find it.

This "tragedy of the commons" endangers the vaccination programme: either because of depletion of the shark population or its drastic reduction. Various remedies for the regulating agency are suggested.

Despite terminology related to a specific problem, this is a starting point to a general theory how to counteract such risks a priori.

A global version of Tarski-Kantorovitch theorem for correspondences

Łukasz Woźny

SGH Warsaw School of Economics
Department of Quantitative Economics
email: lukasz.wozny@sgh.waw.pl

joint work with Łukasz Balbus (U of Zielona Góra), Wojciech Olszewski (Northwestern University) and Kevin Reffett (Arizona State University)

Abstract

For a strong set order increasing (resp., strongly monotone) upper order-der hemicontinuous correspondence $F : A \rightrightarrows A$, where A is a complete lattice (resp., a σ-complete lattice), we provide tight fixed-point bounds for sufficiently large iterations $F^k(a^0)$, starting from any point $a^0 \in A$. Our results, hence, prove a generalization of the Tarski-Kantorovich principle. We provide application of our results to a class of social learning models on networks.

Optimal portfolio selection on the bond market

Dariusz Zawisza

Jagiellonian University
Faculty of Mathematics and Computer Science
email: dariusz.zawisza@uj.edu.pl

Abstract

We assume that our investment strategy is a general signed measure distributed on all real numbers representing time to maturity specifications for different rolling bonds. A nice feature of the model is the possibility to include portfolio consisting of different coupon bonds. The dynamics of the instantaneous interest rate follows a factor model determined by a system of stochastic differential equations. We solve a dynamic consumption–investment problem (joint paper with Szymon Peszat) as well as a dynamic mean-variance problem (joint work with Jakub Trybuła).

[1] Peszat S., Zawisza D., *The investor problem based on the HJM model*, Annales Polonici Mathematici 127 (2021), 241–269.