Thematic section

HSM
Hilbert Spaces Methods

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SCHEDULE OF THE SECTION
Hilbert Spaces Methods

- Monday – September 4th

    16:00–16:30 Antonio Galbis, *Hausdorff Operators on Fock spaces*

    16:30–17:00 Pedro J. Miana, *Hilbertian Hardy-Sobolev spaces on a half-plane*

    coffee break

    17:30–18:00 Paweł Pietrzycki, *Arveson's hyperrigidity*

    18:00–18:30 Wojciech Młotkowski, *On freely quasi-infinitely divisible distributions*

    18:30–19:00 Łukasz Kosiński, *TBA*

- Tuesday – September 5th

    14:30–15:00 Miguel Monsalve-López, *Dunford property for composition operators on H^p-spaces*

    15:00–15:30 Daniel Seco, *Distribution of primes and approximation on weighted Dirichlet spaces*

    15:30–16:00 Wiktor Ejsmond, *A cyclic Fock space of type B*

    coffee break

    16:30–17:00 Marcin Marciniak, *Application of Generalized Gleason Theorem to semifinite factors*

    17:00–17:30 Sergiusz Kużel, *Dual frames and Naimark dilation theorem*

A cyclic Fock space of type B

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Abstract

We introduce a two-parameter function $\phi_{q,s}$ on the infinite hyperoctahedral group, which is a bivariate refinement of the reflection length keeping track of the long and the short reflections separately. We provide a complete characterization of the parameters q, s when the signed reflection function $\phi_{q,s}$ is positive definite and we prove that this condition holds if and only if $\phi_{q,s}$ is an extreme character of the infinite hyperoctahedral group. We construct the corresponding representations as a natural action of the hyperoctahedral group $B(n)$ on the tensor product of n copies of a vector space, which gives a two-parameter analog of the classical construction of Schur–Weyl.

We apply our characterization to construct a cyclic Fock space of type B which generalizes the one-parameter construction in type A found previously by Bożejko and Guta. We also construct a new cyclic Gaussian operator of type B and we relate its moments with the Askey–Wilson–Kerov distribution by using the notion of cycles on pair-partitions, which we introduce here.

Hausdorff Operators on Fock spaces

Antonio Galbis

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Abstract

We discuss boundedness and compactness of Hausdorff operators acting on Fock spaces. The compactness result for Hausdorff operators on the Fock space F_α^∞ is extended to more general Banach spaces of entire functions with weighted sup norms defined in terms of a radial weight. We also include some results on p-summing operators.

[1]   Blasco O., Galbis A., *Boundedness and compactness of Hausdorff operators on Fock spaces*, preprint.

[2]   Bonet J., *Hausdorff operators on weighted Banach spaces of type H^∞*, Complex Analysis and Operator Theory 16 (2022), no.1.

[3]   Galanopoulos P., Stylogiannis G., *Hausdorff operators on Fock Spaces and a coefficient multiplier problem*, Proceedings of the American Mathematical Society 151 (2023), no.7, 3023–3035.

[4]   Tung J., *Taylor coefficients of functions in Fock spaces*, Journal of Mathematical Analysis and Applications 318 (2006), 397–409.

Dual frames and Naimark dilation theorem

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Abstract

Although Naimark dilation theorem was originally stated in 1940, it still finds many important applications in signal processing, computer science, engineering, quantum information theory. The original result, established by Naimark for the case of a generalized resolution of identity [3] was analogized for a Parseval frame by Han and Larson [1]:

Theorem 1. *Let $F_e = \{e_j, \, j \in \mathbb{N}\}$ be a Parseval frame in a Hilbert space K. Then there exists a Hilbert space M and a complementary Parseval frame $F_m = \{m_j, \, j \in \mathbb{N}\}$ in M such that the set of vectors $F_{e \oplus m} = \{e_j \oplus m_j, \, j \in \mathbb{N}\}$ is an orthonormal basis of $H = K \oplus M$.*

A complementary Parseval frame F_m was described in [1] through the identification of H with $l_2(\mathbb{N})$. Such a description of F_m is not always adequate. Since Theorem 1 holds numerous significant applications, it becomes crucial to discover a relatively simple representation of the complementary frame F_m using the original Parseval frame F_e. In this talk, a method for explicitly constructing of F_m in scenarios where the initial frame F_e includes a Riesz basis is presented. These findings are subsequently utilized in the construction of dual frames [2].

[1] Han D., Larson D.R., *Frames, bases and group representations*, Memoirs of the American Mathematical Society 147 (2000), no. 697.

[2] Kamuda A., Kużel S., *On description of dual frames*, Applied and Computational Harmonic Analysis 56 (2021), 351–367.

[3] Naimark M.A., *Spectral functions of a symmetric operator*, Izvestiya Rossiiskoi Akademii Nauk. SSSR Seriya Matematicheskaya 4 (1940), 271–318.

Application of Generalized Gleason Theorem to semifinite factors

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Abstract

The Generalized Gleason Theorem [1] states that for any von Neumann algebra M with no direct summand of type I_2 and any Banach space X each bounded X-valued measure μ on the projection lattice of M extends uniquely to a bounded linear operator from M to X. We consider the case when X is another von Neumann algebra N and μ is a positive measure. Then naturally μ extends to a positive map $\phi : M \to N$. We are focused on the following two problems:

- Can the assumptions be weakened?
- Is it possible to characterize the type of positivity of ϕ such as k-positivity, complete positivity, decomposability, etc. in terms of the measure μ?

It turns out that in both problems rank properties of the measure μ play crucial role. Motivated by the results of [2] and Wigner type theorem for semifinite factors [3] we provide some sufficient conditions on the measure μ for decomposability of the map ϕ.

[1] Bunce L.J., Wright D.M., *The Mackey-Gleason problem*, Bulletin of the American Mathematical Society 26 (1992), p. 288–293.

[2] Marciniak M., *On extremal positive maps between type I factors*, Banach Center Publications 89 (2010), p. 201–221.

[3] Qiana W., Wang L., Wu W., Yuan W., *Wigner-type theorem on transition probability preserving maps in semifinite factors*, Journal of Functional Analysis 276 (2019), p. 1773–1787.

Hilbertian Hardy-Sobolev spaces on a half-plane

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joint work with José E. Galé, Valentin Matache
and Luis Sánchez–Lajusticia

Abstract

In this talk we deal with a scale of reproducing kernel Hilbert spaces $H_2^{(n)}$, $n \geqslant 0$, which are linear subspaces of the classical Hilbertian Hardy space on the right-hand half-plane $\mathbb{C}^+$. They are obtained as ranges of the Laplace transform in extended versions of the Paley-Wiener theorem which involve absolutely continuous functions of higher degree. An explicit integral formula is given for the reproducing kernel $K_{z,n}$ of $H_2^{(n)}$, from which we can find the estimate $\|K_{z,n}\| \sim |z|^{-1/2}$ for $z \in \mathbb{C}^+$. Then composition operators $C_\varphi : H_2^{(n)} \to H_2^{(n)}$, $C_\varphi f = f \circ \varphi$, on these spaces are discussed, giving some necessary and some sufficient conditions for analytic maps $\varphi : \mathbb{C}^+ \to \mathbb{C}^+$ to induce bounded composition operators. These results are included in two joint papers with J.E. Galé, V. Matache and L. Sánchez Lajusticia.

[1] Galé J.E., Matache V., Miana P.J., Sánchez-Lajusticia L., *Hilbertian Hardy-Sobolev spaces on a half-plane*, Journal of Mathematical Analysis and Applications 489 (2020), 124–131.

[2] Galé J.E., Miana P.J., Sánchez–Lajusticia L., *RKH spaces of Brownian type defined by Cesàro–Hardy operators*. Analysis and Mathematical Physics 11 (2021), no. 3, 1–34.

On freely quasi-infinitely divisible distributions

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Abstract

Inspired by the notion of quasi-infinite divisibility (QID), we study the class of freely quasi-infinitely divisible (FQID) distributions on $\mathbb{R}$, which admit the free Lévy-Khintchine-type representation with signed Lévy measure. We prove several properties of the FQID class, some of them in contrast to those of the QID class. For example, a FQID distribution may have negative Gaussian component, and the total mass of its signed Lévy measure may be negative. We provide a characteristic triplet, with the Lévy measure having nonzero negative part, which is at the same time classical and free characteristic triplet.

[1] Hotta I., Młotkowski W., Sakuma N., Ueda Y., *On freely quasi-infinitely divisible distributions*, arXiv:2107.09473.

[2] Lindner A., Pan L., Sato K., *On quasi-infinitely divisible distributions*, Transactions of the American Mathematical Society 370 (2018), no.12, 8483–8520.

Dunford property for composition operators on H^p-spaces

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joint work with E. A. Gallardo-Gutiérrez and F. J. González-Doña

Abstract

The Dunford property (C) for composition operators on H^p-spaces $(1 < p < \infty)$, as well as for their adjoints, is completely characterized within the class of those induced by linear fractional transformations of the unit disc. As a consequence, it is shown that the Dunford property is stable in such a class addressing a particular instance of a question posed by Laursen and Neumann.

Arveson's hyperrigidity

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joint work with Jan Stochel

Abstract

Motivated both by the fundamental role of the classical Choquet boundary in classical approximation theory, and by the importance of approximation in the contemporary theory of operator algebras, Arveson introduced hyperrigidity as a form of approximation that captures many important operator-algebraic phenomena. I will discuss new results on the notion of hyperrigidity such as the characterization of spectral measures, the intertwining theorem, the relationship between the convergence of subnormal operators in weak and strong operator topologies, and new examples of sets of generators that are hyperrigid.

[1] Pietrzycki P., Stochel J., *Subnormal nth roots of quasinormal operators are quasinormal*, Journal of Functional Analysis 280 (2021), no. 12.

[2] Pietrzycki P., Stochel J., *Two-moment characterization of spectral measures on the real line*, Canadian Journal of Mathematics 75 (2022), no. 4.

[3] Pietrzycki P., Stochel J., *On nth roots of bounded and unbounded quasinormal operators*, Annali di Matematica Pura ed Applicata 202 (2022), p. 1313-1333.

[4] Pietrzycki P., Stochel J., *Hyperrigidity: characterizations of spectral measures and convergence of subnormal operators*, in preparation.

Distribution of primes and approximation on weighted Dirichlet spaces

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joint work with Eva A. Gallardo-Gutiérrez

Abstract

We study zero-free regions of the Riemann zeta function ζ related to an approximation problem in the weighted Dirichlet space D_{-2} which is known to be equivalent to the Riemann Hypothesis since the work of Báez-Duarte. We prove, indeed, that analogous approximation problems for the standard weighted Dirichlet spaces D_α when $\alpha \in (-3, -2)$ give conditions so that the half-plane $\{s \in C : \Re(s) > -\frac{\alpha+1}{2}\}$ is also zero-free for ζ. Moreover, we extend such results to a large family of weighted ℓ^p-spaces of analytic functions. As a particular instance, in the limit case $p = 1$ and $\alpha = -2$, we provide a new proof of the Prime Number Theorem.