Thematic section

GTM
Geometry and Topology of Manifolds

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SCHEDULE OF THE SECTION
Geometry and Topology of Manifolds

- Monday – September 4th

  16:00–17:00 Antonio Viruel, *Finite sets containing zero are mapping degree sets*

  coffee break

  17:30–18:15 Maciej Borodzik, *Concordance implies regular homotopy in codimension 2*

  18:15–19:00 Łukasz Michalak, *Reeb graph invariants of Morse functions, manifolds and groups*

- Tuesday – September 5th

  14:30–15:15 Aniceto Murillo, *The rational homotopy type of classifying spaces of homotopy automorphisms*

  15:15–16:00 Anna Gąsior, *Spin-structures on real Bott manifolds with Kaehler structure*

  coffee break

  16:30–17:15 Aleksandra Borówka, *Quaternionic manifolds with rotating circle action*

  17:15–18:00 Pawel Raźny, *A spectral sequence for free isometric lie algebra actions*

- Wednesday – September 6th

  12:00–12:30 Maciej Czarnecki, *Boundary of Hadamard foliations and laminations*

  12:30–13:00 Kacper Grzelakowski, *Triple points on Calabi-Yau threefolds*

  13:00–13:30 Wacław Marzantowicz, *Lefschetz number of equivariant mapping defined in equivariant cohomology theory*

- Thursday – September 7th

  14:30–15:15 Jordi Daura Serrano, *Large Finite group actions on aspherical manifolds*

  15:15–16:00 Martín Saralegui Aranguren, *Some Gysin sequences*

  coffee break

  16:30–17:15 Rafał Lutowski, *Complex Vasquez invariant*

  17:15–18:00 Andreas Zastrow, *The configuration spaces of the Earring Space are aspherical*

Concordance implies regular homotopy in codimension 2

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joint work with Mark Powell and Peter Teichner

Abstract

We introduce immersed Morse theory to prove that concordance implies regular homotopy in codimension 2.

Quaternionic manifolds with rotating circle action

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Abstract

B. Feix [3] (and D. Kaledin [5] independently) showed that there exists a hyperkähler metric on a neighbourhood of the zero section of the cotangent bundle of any real-analytic Kähler manifold. B. Feix provided an explicit construction of its twistor space and showed that any hyperkähler manifold admitting a rotating circle action near its maximal fixed point set arises locally in this way. The construction have been further generalized to hypercomplex manifolds (see Feix [4], Kaledin [6]), quaternionic manifolds (see Borówka, Calderbank [2]) and quaternion-Kähler manifolds (see Borówka [2]). In this talk we will discuss the cases of the construction. Then we will show how to apply it, to obtain a local classification result for quaternionic manifolds with rotating circle action near maximal fixed point set. Finally we will mention connections with c-map.

[1] Borówka A., *Quaternion-Kähler manifolds near maximal fixed point sets of S^1-symmetries*, AMPA 2020

[2] Borówka A., Calderbank D., *Projective geometry and the quaternionic Feix–Kaledin construction*, Transactions of the American Mathematical Society 2019.

[3] Feix B., *Hyperkahler metrics on cotangent bundles*, Journal für die reine und angewandte Mathematik 532 (2001), 33–46.

[4] Feix B., *Hypercomplex manifolds and hyperholomorphic bundles*, Mathematical Proceedings of Cambridge Philosophical Society 133 (2002), 443–457.

[5] Kaledin D., *Hyperkähler metrics on total spaces of cotangent bundles*, in D. Kaledin, M. Verbitsky, Hyperkähler manifolds, Math. Phys. Series **12**, International Press, Cambridge MA, 1999.

[6] Kaledin D., *A canonical hyperkähler metric on the total space of a cotangent bundle*, in Proceeedings of the Second Quaternionic Meeting, Rome (1999), World Scientific, Singapore, 2001.

Boundary of Hadamard foliations and laminations

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Abstract

An Hadamard foliation (or lamination) is a foliation (resp. lamination) of an Hadamard manifold (resp. Hadamard metric space) with all the leaves possesing this property i.e. being connected, simply connected, complete and nonopositively curved.

We shall discuss conditions under which ideal boundaries of leaves laminate the ideal boundary of carrying space and observe how does it work in case of the contracting boundary.

[1] Charney R., Sultan H., *Contracting boundaries of CAT(0) spaces*, Journal of Topology 8 (2015), 93–117.

[2] Czarnecki M., *Hadamard foliations of* H^n, Differential Geometry and its Applications 20 (2004), 357–365.

[3] Czarnecki M., *Umbilical routes along geodesics and hypercycles in the hyperbolic space*, Differential Geometry and its Applications 64 (2019), 47–58.

Large finite group actions on aspherical manifolds

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Abstract

The theory of finite transformation groups studies the symmetries of objects like topological spaces or manifolds by means of finite group actions. Two fundamental question are the following: Given a closed manifold, which finite groups can act effectively on it? Conversely, which topological properties should a closed manifold M have if we know a collection of finite groups actions on M? The answer to these questions in full generality is currently out of reach. One way to simplify them is to study which properties do large finite groups acting on M should fulfil.

In this talk we will show how we can address these questions by studying the Jordan property on the homeomorphism group of closed manifolds or by introducing invariants like the discrete degree of symmetry (see [1] for a recent survey on the topic). We will focus on the case of aspherical manifolds, providing new examples of closed manifolds with Jordan homeomorphism group and computing their discrete degree of symmetry. We would also introduce a theory of iterated finite group actions, which will help us to study rigidity questions on nilmanifolds.

[1] Riera I.M., *Actions of large finite groups on manifolds*, arXiv:2303.07784 (2023).

Spin-structures on real Bott manifolds with Kaehler structure

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Abstract

The main purpose of the article is the problem of existence of spin structures on real Bott manifolds which admit a Kähler structure.

It is known that spin structures on finite quotients of flat tori (flat manifolds) is strictly connected to Sylow 2-subgroups of their holonomy groups. But even knowing that the problem isn't solved for flat manifolds with holonomy group being elementary abelian 2-group, although few years ago significant progress has been made in the case when the action of a finite group on a torus is in a sense diagonal. We take advantage of it and show that when a real Bott manifold admits a Kähler structure then existence of a spin structure on it can be formulated with an easy combinatorial condition.

Triple points on Calabi-Yau threefolds

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Abstract

We discuss the bounds for the number of ordinary triple points on complete intersection Calabi-Yau threefolds in projective spaces and for Calabi-Yau threefolds in weighted projective spaces. In particular we show that in $\mathbb{P}^5$ the intersection of a quadric and a quartic cannot have more than 10 ordinary triple points. We provide examples of complete intersection Calabi-Yau threefolds with multiple triple points. We obtain the exact bound for a sextic hypersurface in $\mathbb{P}[1:1:1:1:2]$ which is 10. We also discuss Calabi-Yau threefolds that cannot admit triple points.

[1] Cynk S., *Hodge numbers of hypersurfaces in $\mathbb{P}^4$ with ordinary triple points*, Advances in Geometry 21 (2021), no. 2, 293–298.

[2] Dolgachev I., *Weighted projective varieties*, Group actions and vector fields (1981), 34–71.

[3] Dolgachev I., *Corrado Segre and nodal cubic threefolds*, In: Casnati G., Conte A., Gatto L., Giacardi L., Marchisio M., Verra A. (eds) From Classical to Modern Algebraic Geometry. Trends in the History of Science. BirkhĂ¤user (2016), 429–450.

[4] Endrass S., Persson U., Stevens J., *Surfaces with triple points*, Journal of Algebraic Geometry 12 (2003), 367–404.

[5] Finkelnberg H., Werner J., *Small resolutions of nodal cubic threefolds*, Indagationes Mathematicae (Proceedings) 92 (1989), no. 2, 185–198.

[6] Fortuna E., Frigerio R., Pardini R., *Projective Geometry Solved Problems and Theory Review*, Springer International Publishing Switzerland, 2016.

[7] Gross M., Popescu S., *Calabi-Yau threefolds and moduli of Abelian Spaces I*, Compositio Mathematica 127 (2001), 169–118.

[8] Kapustka G., Kapustka M., *Primitive contractions of Calabi-Yau threefolds*, Communications in Algebra 37 (2009), no. 2.

[9] Kloosterman R., Rams S., *Quintic threefolds with triple points*, Communications in Contemporary Mathematics 23 (2021), no. 1.

[10] Reid M., *Graded rings and varieties in weighted projective space*, https://homepages.warwick.ac.uk>more>grad, 2002.

[11] Roberts J., *Hypersurfaces with Nonsingular Normalization and Their Double Loci*, Journal of Algebra 53 (1978), 253–267.

[12] Stevens J., *Sextic surfaces with ten triple points*, arXiv:0304060v1 (2003).

[13] van Straten D., *A quintic hypersurface in $\mathbb{P}^4$ with 130 nodes*, Topology 32 (1993), no. 4, 857–864.

Complex Vasquez invariant

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joint work with Anna Gąsior

Abstract

A flat manifold is a closed connected Riemannian manifold with vanishing sectional curvature. By the Auslander-Kurnanishi theorem, every finite group is a holonomy group of some flat manifold. In 1970 Vasquez showed, that for every finite group G there is a natural number $n(G)$ such that every flat manifold X' with holonomy group G is a flat toral extension of a flat manifold X of dimension less than or equal to $n(G)$. In particular this means that we have a fiber bundle

$$T \to X' \to X,$$

where T is a flat torus.

In the talk I will present an analogue of Vasquez number, which is defined for the family compact flat Kähler manifolds. Besides showing some dependencies between real and complex versions of the invariant, I will focus on the problem of the projection map of the bundle to be holomorphic.

Lefschetz number of equivariant mapping defined in equivariant cohomology theory

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joint work with Arturo Espinosa-Baro

Abstract

In late 80-ties of twenty century in the works of M. Atiyah and G. Segal, and later of F. Hirzebruch and T. Höfer, an invariant related to the Euler characteristic of the orbit space had been studied. This invariant was inspired by a paper of the theoretical physicists L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten. In the first two articles it was shown that this invariant is expressed in the terms of equivariant K-theory. Independently, a decade earlier the second author studied a Lefschetz number $\lambda_G(f)$ of equivariant map $f : X \xrightarrow{G} X$ in the K_G^*-theory showing its main properties. The aim of this project is to show that: The Lefschetz number $\lambda_G(f) \in \mathbb{R}(G) \otimes \mathbb{C}$ shares majority of properties of the Euler characteristic type invariant mentioned above generalizing the latter. Its specification for $f = \mathrm{id}_{|X}$ in K_G^*-theory gives previously defined invariant thus extends results of referred works. Applications to study the existence of fixed orbits of maps equivariant with respect of co-finite groups, e.g. the crystallographic group,s is in progress.

Reeb graph invariants of Morse functions, manifolds and groups

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Abstract

The Reeb graph of a Morse function on a closed manifold is obtained by contracting each connected component of its level sets. There are two necessary and sufficient conditions for a finite graph to be realized as the Reeb graph of a Morse function on a given closed manifold: it needs to have the so-called good orientation and its first Betti number cannot exceed the corank of the fundamental group of the manifold. Moreover, any free quotient of this group can be represented as the Reeb epimorphism of a Morse function which is induced on fundamental groups by the quotient map from the manifold to the Reeb graph. It leads to the study of relations between the notions of equivalence of epimorphisms onto free groups, cobordism of systems of hypersurfaces and topological conjugation of Morse functions.

However, the realization of a graph as the Reeb graph of a Morse function is possible only up to a homeomorphism of graphs in general. The minimum number of degree 2 vertices in Reeb graphs of Morse functions is a strong invariant of the topology of the manifold. It has three essentially different lower bounds, which for orientable 3-manifolds are improved by the inequality involving the Heegaard genus. Moreover, another bound is defined in terms of finite presentations of the fundamental group. We use Freiheitssatz, a fundamental fact from one-relator groups, to calculate it in some cases.

The rational homotopy type of classifying spaces of homotopy automorphisms

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joint work with Mario Fuentes and Yves Félix and extracted from [1, 2]

Abstract

We will describe the rational homotopy type of classifying spaces of homotopy automorphisms of nilpotent complexes in terms of certain Lie algebras of derivations.

[1] Fuentes M., Félix Y., Murillo A., *Lie models of homotopy automorphisms monoids and classifying fibrations*, Advances in Mathematics 402 (2022), 1–64.

[2] Fuentes M., Félix Y., Murillo A., *Realization of Lie algebras of derivations and moduli spaces of some rational homotopy types*, arXiv:2206.14124v1 (2022), to appear in Algebraic and Geometric Topology.

A spectral sequence for free isometric lie algebra actions

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Abstract

Assume that (M, g) is a compact Riemannian manifold with a free isometric action of a Lie algebra $\mathfrak{g}$. We present a new spectral sequence, arising from the restriction of the standard filtration of the spectral sequence of the foliation $\mathcal{F}_G$ (see [1]) by orbits of the $\mathfrak{g}$-action to a certain subcomplex of the de Rham complex, which connects the basic cohomology of the foliation, the Lie algebra cohomology of $\mathfrak{g}$ and the de Rham cohomology of M. The construction is a generalization of the Gysin long exact sequence in Sasakian Geometry (see [3]) and is an extension of our prior work [4] on $\mathcal{K}$-structures (see [2]).

[1] Álvarez López J.A., *A finiteness theorem for the spectral sequence of a Riemannian foliation*, Illinois Journal of Mathematics 33 (1989), no. 1, 79–92.

[2] Blair D.E., *Geometry of manifolds with structural group $\mathcal{U}(n)\times\mathcal{O}(s)$*, Journal of Differential Geometry 4 (1970), no. 2, 155–167.

[3] Boyer C.P., Galicki K., *Sasakian Geometry*, Oxford Mathematical Monographs, Oxford University Press, 2007.

[4] Raźny P., *Cohomology of manifolds with structure group $U(n)\times O(s)$*, Geometriae Dedicata 217 (2023), no. 58.

Some Gysin sequences

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Abstract

We are studying a smooth isometric action $\Phi\colon G \times M \to M$ of a Lie group on a manifold M, and our goal is to establish certain connections between the cohomology of M and that of the quotient space M/G.

Which cohomology groups should we consider for M/G?

In the case of an almost free action, the basic cohomology of M/G is sufficient. However, when more complex isotropy subgroups are involved, the basic intersection cohomology of M/G becomes a better adapted tool. In addition to introducing this cohomology, we will conclude the talk by presenting several Gysin sequences that cover specific cases such as $G = \mathbb{R}$, $\mathbb{S}^1$, and $\mathbb{S}^3$.

[1] Royo Prieto J.I., Saralegi Aranguren M., *The Gysin sequence for $\mathbb{S}^3$-actions on manifolds*, Publicationes Mathematicae Debrecen 83 (2013) no. 3, 275–289.

[2] Royo Prieto J.I., Saralegi Aranguren M., *The Gysin braid for S^3-actions on manifolds*, arXiv:2301.09002 (2023).

Finite sets containing zero are mapping degree sets

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joint work with Cristina Costoya and Vicente Muñoz

Abstract

In this lecture, we address several questions which have been raised about $D(M, N)$, the set of mapping degrees between two oriented closed connected manifolds M and N of the same dimension:

$$D(M, N) = \{d \in \mathbb{Z} \mid \exists f : M \to N, \deg(f) = d\}.$$

Neofytidis-Wang-Wang [1, Problem 1.1] discuss the problem of finding, for any set $A \subset \mathbb{Z}$ containing 0, two oriented closed connected manifolds M and N of the same dimension such that $A = D(M, N)$. Note that $0 \in A$ is a necessary condition as the constant map $M \to N$ is of degree zero.

A cardinality argument shows that when A is an infinite set, the problem above is solved in the negative [1, Theorem 1.3].

In contrast, we shall show that given A, any finite set of integers containing 0, there exist (infinitely many) oriented closed connected manifolds M, N such that $A = D(M, N)$. Moreover, the manifolds M, N above can be chosen to be either 3-dimensional, $(4m - 1)$-dimensional for $m \geqslant 4$ or simply connected.

[1] Neofytidis C., Wang S., Wang Z., *Realising sets of integers as mapping degree sets*, to appear in Bulletin of London Mathematical Society.

The configuration spaces of the Earring Space are aspherical

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Abstract

The Earring Space (that during the past 40 years has usually been called "the Hawaiian Earrings") is a subspace of the plane which very much resembles a graph, apart from having not the topology of a graph at one of its points. But due to the accumulation at that point it must be considered as a non-triangulable space. It is known to be aspherical as a one-dimensional space [3, Corl.], a planar space [6, 2], and by having a generalized universal contractible covering [5, Expl.4.15+(U_3)]. Conversely graphs, open subsets of the plane or any surface different from S^1 and $\mathbb{R}P^2$ are known to have also aspherical (ordered and unordered) configuration spaces [4, Corl.2.2], [1, Corl.3.4]. The fact that it is not known whether these results extend to the nearest non-triangulable spaces has been brought to my attention by Daciberg Lima Gonçalves. While it is at the time of writing this abstract it is not clear, whether generalized covering space theory and the action of their deck-transformation groups in this case suffice to extend the classical proofs also in the case of the Earring Space, the talk will describe a more direct proof that the ordered and unordered configuration spaces of the Earring Space are, indeed, aspherical.

[1] Abrams A.D., *Configuration spaces of colored graphs*, Geometriae Dedicata 92 (2002), 185–194.

[2] Cannon J.W., Conner G.R., Zastrow A., *One-dimensional sets and planar sets are aspherical*, Topology and its Applications 120 (2002), no. 1-2, 23–45.

[3] Curtis M.L., Fort M.K., *Homotopy groups of one-dimensional spaces*, Proceedings of the American Mathematical Society 8 (1957), 577–579.

[4] Fadell E., Neuwirth L., *Configuration spaces*, Mathematica Scandinavica 10 (1962), 111–118.

[5] Fischer H., Zastrow A., *Generalized universal covering spaces and the shape group*, Fundamenta Mathematicae 197 (2007), 167–196.

[6] Zastrow A., *Planar sets are aspherical*, Habilitationsschrift, Ruhr-Universität Bochum, 1997-1998.