Thematic section

GCGT

Geometric and Combinatorial Group Theory

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SCHEDULE OF THE SECTION
Geometric and Combinatorial Group Theory

- Monday – September 4th

  16:00–17:00 Joan Porti, *A local approach to Anosov groups*

  coffee break

  17:30–18:15 Michał Marcinkowski, *The* L^1*-metric on* $Diff_0(M, area)$

  18:15–19:00 Karol Duda, *Torsion subgroups of small cancellation group*

- Tuesday – September 5th

  14:30–15:15 Ilya Kazachkov, *On the elementary theory of graph products of groups*

  15:15–16:00 Alexander Zakharov, *Relative order, spectrum of a subgroup and related algorithmic problems in groups*

  coffee break

  16:30–17:15 Oleg Bogopolski, *Exponential equations in acylindrically hyperbolic groups*

  17:15–18:00 Sangrok Oh, *Large-scale geometry of graph* 2*-braid groups*

- Wednesday – September 6th

  12:30–13:15 Piotr Nowak, *Coboundary expanders and Gromov hyperbolicity*

- Thursday – September 7th

  14:00–14:45 Yago Antolín, *Even Artin groups of FC-type*

  15:00–15:45 Motiejus Valiunas, *Biautomatic groups and non-positive curvature*

  coffee break

  16:30–17:15 Jacek Świątkowski, *Trees of graphs as boundaries of hyperbolic groups*

  17:15–18:00 Oihana Garaialde Ocañ, *Hausdorff dimension in profinite groups*

Even Artin groups of FC-type

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joint work with Islam Foniqi

Abstract

In this talk we will review some recent developments on a class of Artin groups that contains and shares many properties with right-angled Artin groups. We will concentrate on parabolic subgroups, and a version of a Tits Alternative for these families of groups.

Exponential equations in acylindrically hyperbolic groups

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joint work with Agnieszka Bier

Abstract

Let $\mathbf{X} = \{x_1, x_2, \dots\}$ be an infinite countable set. An *exponential equation* over a group G is an equation of the form

$$a_1 g_1^{x_{t_1}} a_2 g_2^{x_{t_2}} \dots a_n g_n^{x_{t_n}} = 1, \qquad \text{(B)}$$

where $a_1, g_1, \dots, a_n, g_n$ are elements from G (called *coefficients*) and $x_{t_1}, \dots, x_{t_n} \in \mathbf{X}$ are *variables* (which take values in $\mathbb{Z}$).

We show that if G acts acylindrically on a hyperbolic space, then (B) is equivalent to a finite disjunction of finite systems of easier exponential equations which we call *elementarily loxodromic* and *elliptic*.

In the case where G is hyperbolic relative to a collection of peripheral subgroups $\{H_\lambda\}_{\lambda \in \Lambda}$ equation (B) is equivalent to a finite disjunction of finite systems of equations where each equation is *elementarily loxodromic*, *peripheral*, or *finitary*. As corollary, we prove in this case that the solution set of any exponential equation over G is $\mathbb{Z}$-semilinear if and only if the solution set of any exponential equation over every H_λ, $\lambda \in \Lambda$, is $\mathbb{Z}$-semilinear. This generalizes the result of Lohrey [1] that the $\mathbb{N}$-solution set of any exponential equation over a hyperbolic group is semilinear. Note that $\mathbb{Z}$-semilinearity is closely related to definability in the weak Presburger arithmetic.

A simple version of another our result says that if G is a finitely generated acylindrically hyperbolic group and Y is any finite generating set of G, then there exists a number $C > 0$ such that for any exponential equation (B) with generalized loxodromic $g_1, \dots, g_n$ and variables $x_1, \dots, x_n$, if this equation has a solution, then there exists a solution $(k_1, \dots, k_n)$ such that

$$\max_{i=1,\dots,n} |k_i| \leqslant \Big(\sum_{i=1}^{n} |a_i|_Y + \sum_{i=1}^{n} |g_i|_Y \Big) \cdot Cn.$$

This result generalizes and improves the main result in [2] where exponential equations over hyperbolic groups were considered. Finally, we introduce exponential-elementary and exponential-existential theories of groups.

[1] Lohrey M., *Knapsack in hyperbolic groups*, Journal of Algebra 545 (2020), no 1, 390–415.

[2] Myasnikov A., Nikolaev A., Ushakov A., *Knapsack problems in groups*, Mathematics of Computations 84 (2015), no. 292, 987–1016.

Hypercubical groups: from RAAGs to knot theory and beyond

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Abstract

Right-angled Artin groups form an outstanding and broadly studied class of groups due to their remarkable group-theoretic, cohomological, geometric, and combinatorial properties. To each RAAG one can associate a non-positively curved cubical complex, namely the Salvetti complex, whose universal covering is a CAT(0) cube complex and therefore it is contractible. Mimicking this phenomenon, we define the class of hypercubical groups as the class of those finitely generated groups G for which a certain cubical complex, namely the hypercubical complex of G, is contractible. The aim of this talk is to introduce such family of groups and show different cases where this phenomenon shows up. In particular, we will focus on a generalization of RAAGs and on a family of groups generalizing the link group of the Borromean rings.

Torsion subgroups of small cancellation group

Karol Duda

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Abstract

We prove that torsion subgroups of groups defined by, $C(6)$, $C(4)$-$T(4)$ or $C(3)$-$T(6)$ small cancellation presentations are finite. This follows from more general results about locally elliptic action on small cancellation complexes.

[1] Duda K., *Torsion subgroups of small cancellation groups*, arXiv:2112.01912 (2021).

On the elementary theory of graph products of groups

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Abstract

When studying the model theory of groups, it is natural to ask which group-theoretic constructions preserve the elementary theory. In 1959, Feferman and Vaught studied the first-order properties of direct products and showed, in particular, that the direct products of elementarily equivalent groups are elementarily equivalent. In contrast, invariance of the elementary equivalence for free products of groups was a long-standing conjecture which was recently solved by Sela (2017).

In this talk, we will first address the converse question: given two elementary equivalent free products of groups (or more generally, graph product of groups), when are the factors elementarily equivalent? We discuss some sufficient conditions and use our results to describe finitely generated groups elementarily equivalent to RAAGs whose underlying graph is a transitive forest.

The L^1-metric on $Diff_0(M, area)$

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joint work with M. Brandenbursky and E. Shelukhin

Abstract

Let M be a compact Riemannian manifold. There are a number of interesting metrics on the group of volume preserving diffeomorphisms of M, among them the L^1-metric. If M is an $(n > 2)$-dimensional disc, then the diameter of $Diff_0(M, vol)$ with the L^1-metric is finite by the celebrated result of A. Shnirelman. In the 2-dimensional case the situation is very different. In this talk I will show how to use braids to estimate the L^1-metric on $Diff_0(M, area)$ where M is a compact surface. As an application we construct many L^1-Lipschitz quasimorphisms on $Diff_0(M, area)$ and show that all right-angled Artin groups embed quasi-isometrically into $Diff_0(M, area)$.

Coboundary expanders and Gromov hyperbolicity

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joint work with Dawid Kielak

Abstract

I will discuss the concept of higher-dimensional expanders as a generalization of expander graphs. I will show that for a tower of residual finite coverings of a compact manifold, coboundary expansion in a certain dimension forces the fundamental group to be hyperbolic.

Hausdorff dimension in profinite groups

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Abstract

Hausdorff dimension is a tool to measure the size of fractal objects and subsets of a metric space. In [1], Y. Barnea and A. Shalev studied this notion in the context of profinite groups which are inverse limits of finite groups. In the first part of this talk, we explain some of the open questions and main results in this area (compare [1] and [2]). If time permits, we will explore a new result for finitely generated regular branch groups.

[1] Barnea Y., Shale A., *Hausdorff dimension, pro-p groups, and Kac-Moody algebras*, Transactions of the American Mathematical Society 349 (1997), no. 12, 5073–5091.

[2] Klopsch B., Thillaisundaram A., Zugadi-Reizabal A., *Hausdorff dimensions in p-adic analytic groups*, Israel Journal of Mathematics 231 (2019), 1–23.

[3] Shalev A., *Lie methods in the theory of pro-p groups, In New horizons in pro-p groups*, Birkhäuser Boston, Boston, 2000.

Large-scale geometry of graph 2-braid groups

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Abstract

Graph n-braid groups or n-braid groups over graphs are the fundamental groups of configuration spaces on graphs. Unlike configuration spaces on higher dimensional spaces, there is a discrete version of configuration space on a graph, which is a locally CAT(0) cube complex, and in particular, graph braid groups act geometrically on CAT(0) cube complexes. In this talk, using quasi-isometry invariants of CAT(0) square complexes, called intersection complexes, we will talk about the quasi-isometric classification of 2-braid groups over a special kind of graphs.

A local approach to Anosov groups

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joint work with M. Kapovich and B. Leeb

Abstract

Given finitely many elements of a semi-simple Lie group G of non compact type, I give a sufficient criterion so that they span an Anosov group (in particular discrete and word hyperbolic). The criterion is based on the action on the symmetric space G/K. As an application, I discuss the algorithmic recognizability of Anosov groups.

Trees of graphs as boundaries of hyperbolic groups

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Abstract

Despite many years of extensive study, various basic aspects of hyperbolic groups are still far from being well understood. For example, surprisingly little is still known about topological classification of Gromov boundaries of hyperbolic groups. We present here a new result in this direction (joint with Nima Hoda from Cornell University), which can be viewed as a complete classification of connected Gromov boundaries of the simplest possible form in topological dimension 1 (other that the cirle S^1). The corresponding spaces are called *regular trees of 2-connected graphs*, and they form a class of spaces which have particularly many local cut points (so that the Sierpinski curve and the Menger curve are beyond this class). We characterize completely, in algebraic terms, the family of all those hyperbolic groups G whose Gromov boundary ∂G is in this class. The latter characterization is obtained in terms of some features of the so called canonical JSJ splitting of G (along its 2-ended subgroups).

As a matter of fact, we indicate also a subclass in the above mentioned class of trees of graphs, called *trees of* Θ*-graphs*, which consists of even simpler spaces. We characterize also those hyperbolic groups G whose Gromov boundaries ∂G belong to this subclass. This exhibits some gradation of complexity among 1-dimensional connected Gromov boundaries ∂G, as well as among the corresponding groups G (in the latter case this gradation has a rather transparent algebraic flavour).

In order to understand the precise statements of our results, recall that the factors of the canonical JSJ splitting of a hyperbolic group fall into two separate classes. The well understood class of factors is that of so called *flexible factors*, and the more complex one is that of so called *rigid factors.* As we explain, rigid factors can be either isolated from each other (in some precise sense) be means of flexible factors, or they can form some groups of non-isolated rigid factors that we call *rigid cluster factors*. Recall also that *trees of graphs* form a natural class of topological spaces larger than that of regular trees of 2-connected graphs, and that a graph is 2-connected if it is connected and contains no cut vertex. A *thick* Θ*-graph* is a graph having precisely 2 vertices and at least 3 edges each of

which connects those two vertices. Recall finally that a hyperbolic group G is virtually Fuchsian iff its Gromov boundary ∂G is homeomorphic to the circle S^1. Our main results read then as follows.

Theorem A. *Let G be a 1-ended hyperbolic group which is not virtually Fuchsian. Then the following conditions are equivalent:*

(1) *∂G is homeomorphic to a tree of graphs;*
(2) *∂G is homeomorphic to a regular tree of 2-connected graphs;*
(3) *each rigid cluster factor of G is virtually free.*

Theorem B. *Let G be a 1-ended hyperbolic group that is not virtually Fuchsian. Then the following conditions are equivelent:*

(1) *∂G is homeomorphic to a tree of thick Θ-graphs;*
(2) *∂G is homeomorphic to a regular tree of thick Θ-graphs;*
(3) *G has no rigid factor in its canonical JSJ splitting.*

Biautomatic groups and non-positive curvature

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joint work with Sam Hughes

Abstract

Biautomatic groups, introduced in the early 1990s, arose as groups explaining formal language-theoretic aspects of word-hyperbolic groups. Many classes of non-positively curved finitely generated groups, such as hyperbolic, virtually abelian, cocompactly cubulated and Coxeter groups, are known to be biautomatic.

In this talk, I will give a brief introduction to the class of biautomatic groups and its relation to various classes of non-positively curved groups. Namely, I will outline some arguments showing that certain CAT(0) groups are not subgroups of biautomatic groups, that certain hierarchically hyperbolic groups are not biautomatic, and/or that the latter groups satisfy a weaker property of being *asynchronously* automatic.

Relative order, spectrum of a subgroup and related algorithmic problems in groups

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joint work with Jordi Delgado and Enric Ventura

Abstract

I will first survey some results about algorithmic problems for subgroups, such as subgroup membership problem. Then I will focus on the notions of relative order and spectrum of subgroups, and related algorithmic problems. The relative order of an element with respect to a subgroup is the minimal positive power in which the element belongs to the subgroup (and 0 if such a power does not exist), and the spectrum of a subgroup is the set of all possible relative orders with respect to that subgroup. I will present both positive and negative results on the computability of spectrum in different groups.

[1] Delgado J., Ventura E., Zakharov A., *Relative order and spectrum in free and related groups*, to appear in Communications in Contemporary Mathematics