Thematic section

DGGA

Differential Geometry and Geometric Analysis

ORGANIZERS:

Maciej Czarnecki (Uniwersytet Łódzki)

Miguel Dominguez Vazquez (Universidade de Santiago de Compostela)

Jacek Jezierski (Uniwersytet Warszawski)

Miguel Sanchez Caja (Universidad de Granada)

SCHEDULE OF THE SECTION
Differential Geometry and Geometric Analysis

- Monday – September 4th

  16:30–17:00 Cristina Sardón, *A geometric journey through the Hamilton-Jacobi theory*

  coffee break

  17:30–18:00 Javier de Lucas, *A characterisation of relative stability in cosymplectic Hamiltonian systems*

  18:00–18:30 Katja Sagerschnig, *Conformal structures with G_2-twistor distribution*

  18:30–19:00 Rouzbeh Mohseni, *Twistors of foliated manifolds*

- Tuesday – September 5th

  14:30–15:00 Mikołaj Rotkiewicz, *Higher order algebroids and representations up to homotopy*

  15:00–15:30 Miguel Ángel Javaloyes, *Finsler spacetimes and its applications to cosmology and wildfire propagation*

  15:30–16:00 Zdeněk Dušek, *Geodesic orbit Finsler (α, β) metrics*

  coffee break

  16:30–17:00 Teresa Arias-Marco, *Mixed Steklov problems on surfaces*

  17:00–17:30 Wojciech Domitrz, *On singularities of the Gauss map components of surfaces in $\mathbb{R}^4$*

  17:30–18:00 Tomasz Zawadzki, *Variational problems and methods in extrinsic geometry of distributions*

- Wednesday – September 6th

  12:00–12:30 Robert Wolak, *Hard Lefschetz Property for isometric flows and S^3–actions*

  12:30–13:00 Antoni Pierzchalski, *Pairs of foliations and a conformal invariant*

  13:00–13:30 José M. Manzano, *On the convergence of minimal graphs and the prescribed mean curvature equation*

- Thursday – September 7th

  14:00–14:30 Piotr Mormul, *From Engel and Cartan to monsters in algebraic and differential geometry*

  14:30–15:00 Kamil Niedziałomski, *An integral formula for G–structures*

  15:00–15:30 Roberto Rubio, *New geometric structures on 3-manifolds through generalized geometry*

  15:30–16:00 Arman Taghavi-Chabert, *Perturbations of Fefferman spaces over CR three-manifolds*

  coffee break

  16:30–17:00 Alberto Rodríguez Vázquez, *Totally geodesic submanifolds and homogeneous spheres*

  17:00–17:30 Daniel Ballesteros Chávez, *On the Isometric embeddings of spheres into de Sitter space*

  17:30–18:00 W. Kryński, *On two constructions in path geometry: dancing and chains*

Mixed Steklov problems on surfaces

Teresa Arias-Marco

University of Extremadura
Department of Mathematics
email: ariasmarco@unex.es

joint work with E. B. Dryden, C. S. Gordon,
A. Hassannezhad, A. Ray, E. Stanhope

Abstract

The talk will focus the attention in the study of eigenvalue asymptotics for mixed-Steklov problems on Riemannian surfaces with Lipschitz boundary of arbitrary genus and arbitrary number of boundary components.

The results obtained in [2] can be viewed as an extension of the result of Girouard, Parnovski, Polterovich and Sher [3] on the full asymptotics of the Steklov spectrum for Riemannian surfaces with smooth boundary, and the result of the authors [1] on the full asymptotics of the Steklov spectrum for orbisurfaces.

[1] Arias-Marco T., Dryden E. B., Gordon C. S., Hassannezhad A., Ray A., Stanhope E., *Spectral geometry of the Steklov problem on orbifolds*, International Mathematics Research Notices 1 (2019), 90–139.

[2] Arias-Marco T., Dryden E. B., Gordon C. S., Hassannezhad A., Ray A., Stanhope E., *Applications of possibly hidden symmetry to Steklov and mixed Steklov problems on surfaces*, arXiv:2301.09010 (2023).

[3] Girouard A., Parnovski L., Polterovich I., Sher D. A., *The Steklov spectrum of surfaces: asymptotics and invariants*, Mathematical Proceedings of Cambridge Philosophical Society 157 (2014), 379–389.

On the Isometric embeddings of spheres into de Sitter space

Daniel Ballesteros Chávez

Silesian University of Technology, Dep. Applied Mathematics
Poznań University of Technology, Institute of Mathematics
email: daniel.ballesteros-chavez@polsl.pl

Abstract

The Weyl problem is a classical question of the isometric embedding of a positively curved 2-sphere in the Euclidean 3-space. It was considered by Hermann Weyl in 1916, and its resolution by Louis Nirenberg in 1953 led to major advances in the theory of nonlinear differential equations of elliptic type.

In this talk we consider (spacelike) isometric embeddings of a metric on the sphere into de Sitter space, with a suitable curvature restriction. We present an a priori estimate for the mean curvature H of such spacelike hypersurface and some recent developments in these directions.

[1] Ballesteros-Chávez D., Klingenberg W., Lambert B., *Weyl estimates for spacelike hypersurfaces in de Sitter space*, Pacific Journal of Mathematics 320 (2022), no. 1, p. 1–11.

[2] Li C., Wang Z., *The Weyl problem in warped product space*, Journal of Differential Geometry 114 (2020), no. 2, p. 243–304.

On singularities of the Gauss map components of surfaces in $\mathbb{R}^4$

Wojciech Domitrz

Warsaw University of Technology
email: wojciech.domitrz@pw.edu.pl

joint work with L. I. Hernández-Martínez and F. Sánchez-Bringas

Abstract

The Gauss map of a generic immersion of a smooth, oriented surface into $\mathbb{R}^4$ is an immersion. But this map takes values on the Grassmanian of oriented 2-planes in $\mathbb{R}^4$. Since this manifold has a structure of a product of two spheres, the Gauss map has two components that take values on the sphere. We study the singularities of the components of the Gauss map and relate them to the geometric properties of the generic immersion. Moreover, we prove that the singularities are generically stable, and we connect them to the contact type of the surface and $\mathcal{J}$-holomorphic curves with respect to an orthogonal complex structure $\mathcal{J}$ on $\mathbb{R}^4$. Finally, we get some formulas of Gauss-Bonnet type involving the geometry of the singularities of the components with the geometry and topology of the surface.

Geodesic orbit Finsler (α, β) metrics

Zdeněk Dušek

Institute of Technology and Business in České Budějovice
Department of Informatics and Natural Sciences
email: zdusek@mail.vstecb.cz

part of this work is a joint work with Teresa Arias-Marco

Abstract

Geodesic lemma for homogenenous Finsler (α, β) metrics F is formulated in terms of the underlying Riemannian metric α and the one-form β. The existence of a particular reductive decomposition is described for easy construction of Finslerian geodesic graph, in a suitable group extension. As a consequence, it is proved that for underlying geodesic orbit Riemannian metric α, all Finsler (α, β) metrics F are also geodesic orbit metrics. An alternative construction of Finslerian geodesic graph for naturally reductive underlying Riemannian metric α is also described. The relation of Riemannian geodesic graphs with Finslerian geodesic graphs is illustrated with explicit constructions on spheres. As a corollary, geodesic orbit Finsler (α, β) metrics F on spheres are determined.

[1] Arias-Marco T., Dušek Z., *Geodesic graphs for geodesic orbit Finsler (α, β) metrics on spheres*, arXiv:2303.09368 (2023).

[2] Dušek Z., *Geodesic orbit Finsler (α, β) metrics*, European Journal of Mathematics 9 (2023), no. 9.

Finsler spacetimes and its applications to cosmology and wildfire propagation

Miguel Ángel Javaloyes

University of Murcia
Department of Mathematics
email: majava@um.e

Abstract

We will first show how Finsler spacetimes naturally appear as a tool to solve the time-dependent Zermelo problem in a manifold M, or more generally, the problem of finding the shortest trajectory in time when the velocity is prescribed at any direction and any instant of time, namely, the velocity is a function of the direction and the time. It turns out that the shortest trajectories are the projections to M of lightlike geodesics in the non-relativistic spacetime $\mathbb{R}\times M$, where the first coordinate is the absolute time. These findings can be applied to wildfire propagation models as the velocity of the fire in every direction and instant of time is prescribed, namely, it depends on the wind, the slope, the vegetation, humidity... so the propagation of the fire can be obtained computing the orthogonal lightlike geodesics to the firefront. On the other hand, Finsler spacetimes can be used as cosmological models in situations with a certain degree of anisotropy. We will discuss the meaning of the stress-energy tensor in this context and some proposals for Einstein field equations.

[1] Javaloyes M.A., Pendás-Recondo E., Sánchez M., *Applications of cone structures to the anisotropic rheonomic Huygens' principle*, Nonlinear Analysis 209 (2021).

[2] Javaloyes M.A., Pendás-Recondo E., Sánchez M., *A general model for wildfire propagation with wind and slope*, SIAM Journal of Applied Algebra and Geometry 7 (2023), no. 2, 414–439.

[3] Javaloyes M.A., Sánchez M., *On the definition and examples of cones and Finsler spacetimes.* RACSAM 114 (2020), no. 30.

[4] Javaloyes M.A., Sánchez M., Villaseñor F.F., *On the significance of the Stress-Energy tensor in Finsler spacetimes*, Universe, 8 (2022), no. 2.

[5] Javaloyes M.A., Sánchez M., Villaseñor F.F., *The Einstein-Hilbert-Palatini formalism in pseudo-Finsler geometry*, arXiv:2108.03197 (2021), to appear in Advances in Theoretical and Mathematical Physics.

A characterisation of relative stability in cosymplectic Hamiltonian systems

Javier de Lucas

University of Warsaw
Department of Mathematical Methods in Physics
email: javier.de.lucas@fuw.edu.pl

Abstract

In short, a relative equilibrium point of a differential equation in a manifold is a point of the manifold whose evolution is described by a symmetry of the differential equation. This talk is concerned with the stability of relative equilibrium points of time-dependent Hamilton equations in symplectic manifolds. More specifically, we aim to analyse the behaviour of trajectories close to solutions evolving via symmetries of time-dependent Hamilton equations. Since relative equilibrium points are not, in general, equilibrium points, standard stability techniques are not available for their study. Other classical techniques to study relative equilibrium points of Hamiltonian systems, like the energy-momentum methods [2], are not enough either since they only cope with time-independent Hamiltonian systems on symplectic manifolds. To generalise the energy-momentum method to a time-dependent realm, we will give a geometric approach to time-dependent Hamiltonian systems through the so-called cosymplectic structures. Next, a new generalisation of the energy-momentum method via cosymplectic geometry will be devised [1]. As an application, the study of restricted circular three-body problems, Lagrange points, and other related physical problems will be accomplished. A special type of cosymplectic-symplectic reduction will be developed and applied.

[1] de Lucas J., Maskalaniec A., Zawora B.M., *A cosymplectic energy-momentum method with applications*, arXiv:2302.05827 (2023).

[2] Marsden J.E., Simo J.C., *The energy momentum method*, Acta Academiae Scientiarum Taurinensis 1 (1988), 245–268.

On the convergence of minimal graphs and the prescribed mean curvature equation

José M. Manzano

Universidad de Jaén
Departamento de Matemáticas
email: jmprego@ujaen.es

Abstract

We will see that the prescribed mean curvature equation for spacelike graphs in Lorentz–Minkowski space $\mathbb{L}^3$ can be transformed into the minimal surface equation in $\mathbb{R}^3$ endowed with a certain Riemannian metric with an unit Killing vector field. We will introduce the theory of divergence lines to see that we can produce an entire minimal graph in the Riemannian setting as a limit of graphs over disks. This gives entire spacelike graphs in $\mathbb{L}^3$ with prescribed mean curvature $H \in \mathcal{C}^\infty(\mathbb{R}^2)$ provided that H and ∇H are bounded. This approach also allows us to prescribe the normal of the entire spacelike graph at one point.

[1] Del Prete A., Lee H., Manzano J.M., *A duality for prescribed mean curvature graphs in Riemannian and Lorentzian Killing submersions*, preprint (2023).

[2] Del Prete A., Manzano J.M., Nelli B., *The Jenkins–Serrin problem in 3-manifolds with a Killing vecto field*, preprint (2023).

Twistors of foliated manifolds

Rouzbeh Mohseni

Jagiellonian University in Krakow
Institute of Mathematics
email: rouzbeh.iii@gmail.com

Abstract

Let M^{2n} be an even-dimensional Riemannian manifold, the twistor space $Z(M)$ is the parametrizing space for compatible almost complex structures on M. It is a bundle over M, with fiber $SO(2n)/U(n)$ and is equipped with two almost complex structures $J^{\pm}$, where J^{+} can be integrable but J^{-} is never integrable, however, it still is important as will be discussed.

This talk is based on joint work with R. A. Wolak [1], in which, the theory of twistors on foliated manifolds is developed and some of the works that we did afterward. We construct the twistor space of the normal bundle of a foliation. It is demonstrated that the classical constructions of the twistor theory lead to foliated objects and permit to formulate and prove foliated versions of some well-known results on holomorphic mappings. Since any orbifold can be understood as the leaf space of a suitably defined Riemannian foliation we obtain orbifold versions of the classical results as a simple consequence of the results on foliated mappings.

[1] Mohseni R., Wolak R.A., *Twistor spaces on foliated manifolds*, International Journal of Mathematics 32 (2021), no. 8.

From Engel and Cartan to monsters in algebraic and differential geometry

Piotr Mormul

University of Warsaw
Institute of Mathematics
email: mormul@mimuw.edu.pl

Abstract

The aim of the proposed talk is: **firstly** to give an overview of a long tree (not a linear sequence!) of works on the local geometry of flag-like and multi-flag-like distributions in the tangent bundles to manifolds. From classical works of Engel [2], von Weber [11] and E. Cartan, via [a non-exhaustive list:] Semple, Gaspar [3], Kumpera-Rubin [4], Lejeune-Jalabert, Laumond-Risler-Jean, Adachi, Yamaguchi, earlier works of the author, Montgomery-Zhitomirskii [5], Kennedy et al [1], Montgomery-Zhitomirskii [5], till the actual investigations still in progress like [9] and [10].

Secondly to briefly discuss kinematical visualisations of the mentioned objects (so-called car + trailers systems in 2D and articulated arms systems, or spececraft + satellites systems in 3D).

Thirdly to focus on still open issues in the local classification of singularities of 1-flags (i.e., Goursat flags) and of special multi-flags (sometimes also called ‘Goursat multi-flags’).

And, time permitting, **fourthly**, to briefly address two global aspects of the discussed structures: the so-called *Engel geometry* and Mormul’s 2016 conjecture about Goursat structures on closed 5-manifolds.

Why [special] flags? A kind of an answer can be found in the pioneering von Weber’s 1898 work [11]. He started that work with an ambitious plan to go far beyond the classical Frobenius theorem and to locally describe *all* geometric distributions. And he eventually settled for the condition now known as ‘Goursat condition’ (i.e., he settled for the 1-flags).

As for special multi-flags, they started to be extensively dealt with by Kumpera-Rubin around the year 2000, [4]. The – still actual – state of the art is, in Kennedy et al 2017’, [1], words, as follows: "we seem to be very far from a full understanding of where and why moduli occur".

[1] Colley S., Castro A., Kennedy G., Shanbrom C., *A Coarse Stratification of the Monster Tower*, Michigan Mathematical Journal 66 (2017), 855–866.

[2] Engel F., *Zur Invariantentheorie der Systeme von Pfaff'schen Gleichungen*, Berichte Ges. Leipzig, Math-Phys. Classe XLI (1889), 157–176.

[3] Gaspar M., *Sobre la clasificacion de sistemas de Pfaff en bandera*, in: Proceedings of 10th Spanish-Portuguese Conference on Math., University of Murcia (1985), 67–74.

[4] Kumpera A., Rubin J.L., *Multi-flag systems and ordinary differential equations*, Nagoya Mathematical Journal 166 (2002), 1–27.

[5] Montgomery R., Zhitomirskii M., *Points and Curves in the Monster Tower*, Memoirs of the American Mathematical Society 956 (2010).

[6] Mormul P., *Goursat flags: classification of codimension-one singularities*, Journal of Dynamical and Control Systems 6 (2000), 311–330.

[7] Mormul P., *Multi-dimensional Cartan prolongation and special k-flags*, Banach Center Publications 65 (2004), 157–178.

[8] Mormul P., *Singularity classes of special 2-flags*, SIGMA 5 (2009), no. 102.

[9] Mormul P., *Monster towers from differential and algebraic viewpoints*, Journal of Singularities 25 (2022), 331–347.

[10] Mormul P., *Singularities of special multi-flags*, to appear.

[11] von Weber E., *Zur Invariantentheorie der Systeme Pfaff'scher Gleichungen*, Berichte Ges. Leipzig, Math-Phys. Classe L (1898), 207–229.

An integral formula for G–structures

Kamil Niedziałomski

University of Lodz
Department of Mathematics and Computer Science
email: kamil.niedzialomski@wmii.uni.lodz.pl

Abstract

Equipping an n–dimensional manifold M with a Riemannian metric g is equivalent to the reduction of a frame bundle $L(M)$ to the orthogonal frame bundle $O(M)$, i.e. to the action of a structure group $O(n)$. Assuming moreover that M is oriented we can consider the bundle $SO(M)$ of oriented orthonormal frames. Existence of additional geometric structure can be considered as a reduction of a structure group $SO(n)$ to a certain subgroup G. For example, almost Hermitian structure gives $U(\frac{n}{2})$–structure, almost contact metric structure is just a $U(\frac{n-1}{2}) \times 1$–structure, etc.

If ∇ is the Levi–Civita connection of (M, g) we may measure the defect of ∇ to be a G–connection. This leads to the notion of an intrinsic torsion. If this $(1, 2)$–tensor vanishes (in such case we say that a G–structure is integrable) then ∇ is a G–connection, which implies that the holonomy group is contained in G. We may classify non–integrable geometries by finding the decomposition of the space of all possible intrinsic torsions into irreducible G–modules. This approach was initiated by Gray and Hervella for $U(\frac{n}{2})$–structures [3] and later considered for other structures by many authors. Each, so–called, Gray–Hervella class, gives some restrictions on the curvature.

One possible approach to curvature restrictions on compact G–structures can be achieved by obtaining integral formulas relating considered objects. This has been firstly done, in a general case, by Bor and Hernández Lamoneda [1]. They uses Bochner–type formula for forms being stabilizers of each considered subgroup in $SO(n)$. They obtained integral formulas for $G = U(\frac{n}{2}), SU(\frac{n}{2}), G_2$ and Spin_7 and continued this approach for $Sp(n)Sp(1)$ in [2]. The case $G = U(\frac{n-1}{2}) \times 1$ has been studied later in [4] by other authors.

We show how mentioned formulas can be obtained in a different way. The motivation comes from the work of Walczak [5], where the author considered almost product structures, i.e., complementary orthogonal distributions. The nice feature of our approach is that the main integral formula

$$\frac{1}{2}\int_M s_{\mathfrak{g}^\perp} - s^{\mathrm{alt}}_{\mathfrak{g}^\perp} \,\mathrm{vol}_M = \int_M |\chi|^2 + |\xi^{\mathrm{alt}}|^2 - |\xi^{\mathrm{sym}}|^2 \,\mathrm{vol}_M.$$

is valid for any G–structure on closed M for compact $G \subset SO(n)$. Let us roughly describe the approach and all used objects in this formula. We consider, so–called, characteristic vector field $\chi = \sum_i \xi_{e_i} e_i$ induced by the intrinsic torsion ξ and calculate its divergence. ξ^{alt} and ξ^{sym} denote the skew–symmetric and symmetric components of ξ, $\xi^{\mathrm{alt}}_X Y = \frac{1}{2}(\xi_X Y - \xi_Y X)$, $\xi^{\mathrm{sym}}_X Y = \frac{1}{2}(\xi_X Y + \xi_Y X)$, whereas, $s_{\mathfrak{g}^\perp}$ and $s^{\mathrm{alt}}_{\mathfrak{g}^\perp}$ are, in a sense, $\mathfrak{g}^\perp$ components of a scalar curvature. Here, $\mathfrak{g}^\perp$ is the orthogonal complement of the Lie algebra $\mathfrak{g}$ of a Lie group G inside the Lie algebra $\mathfrak{so}(n)$ of the Lie group $SO(n)$. For some Gray–Hervella classes the characteristic vector field vanishes, and then we get point–wise formula relating an intrinsic torsion with a curvature.

We derive explicit integral formulas in the following cases:

- almost Hermitian structures,
- special almost Hermitian structures,
- almost contact metric structures,
- G_2 structures,
- Spin(7) structures.

In the way described above we recover many well known relations. Let us state some of the consequences of the main integral formula (without definitions of used objects):

1. Assume (M, g, J) is closed Hermitian manifold of Gray–Hervella type $\mathcal{W}_4$ such that $s = s^*$, where s is a scalar curvature and s^* is a $*$–scalar curvature. Then M is Kähler (compare [1]).
2. On a closed $SU(n)$–structure of type $\mathcal{W}_1 \oplus \mathcal{W}_5$ we have $\int_M s = 5\int_M s^*$.
3. Assume M is a G_2–structure of type $\mathfrak{X}_1 \oplus \mathfrak{X}_2 \oplus \mathfrak{X}_3$ induced by an endomorphism T. Then, the scalar curvature is given by the formula
$$\frac{1}{6}s = -\frac{3}{2}i_0(T) + 6\sigma_2(T),$$
where $i_0(T)$ and $\sigma_2(T)$ are quadratic invariants of T.
4. The scalar curvature of an Spin(7)–structure induced by the Lee form θ and Λ^3_{48} component of $\delta\Phi$ equals
$$s = \frac{21}{8}|\theta|^2 - \frac{1}{2}|(\delta\Phi)_{48}|^2 + \frac{7}{2}\operatorname{div}\theta.$$

[1] Bor G., Hernández L.L., *Bochner formulae for orthogonal G-structures on compact manifolds*, Differential Geometry and its Applications 15 (2001), no. 3, 265–286.

[2] Bor G., Hernández L.L., *A Bochner formula for almost-quaternionic-Hermitian structures*, Differential Geometry and its Applications 21 (2004), no. 1, 79–92.

[3] Gray A., Hervella L., *The Sixteen Classes of Almost Hermitian Manifolds and Their Linear Invariants*, Annali di Matematica Pura ed Applicata 123 (1980) no. 4, 35–58.

[4] González-Dávila J.C., Martin C. F., *Harmonic almost contact structures via the intrinsic torsion*, Israel Journal of Mathematics 181 (2011), 145–187.

[5] Walczak P., *An integral formula for a Riemannian manifold with two orthogonal complementary distributions*, Colloquium Mathematicum 58 (1990), no. 2, 243–252.

Pairs of foliations and a conformal invariant

Antoni Pierzchalski

University of Lodz
Faculty of Mathematics and Computer Science
email: antoni.pierzchalski@wmii.uni.lodz.pl

joint work with Jerzy Kalina and Wojciech Kozłowski

Abstract

Pairs of mutually orthogonal complementary foliations of a Riemanian manifold (M, g) appear naturally in geometry. They appear also in physics. For M being a domain in $\mathbb{R}^3$ let $u : M \to \mathbb{R}$ be the potential of an electric or magnetic field in M. Then the family of equipotential surfaces of u (the first foliation) is orthogonal to the family of lines of the force field (the other foliation). The situation generalizes to pairs of orthogonal foliations defined as the level sets of of a pair of so called conjugate submersions on an arbitrary Riemannian manifold M. The properties of such pairs have been investigated in the context of their conformal capacity in [1] and [2]. Let us also note that pairs of orthogonal distributions, and so, in particular, pairs of foliations, were also investigated by variational methods in the class of Riemannian connections on (M, g) in [4]. Here the geometry of a pair of mutually orthogonal complementary foliations will be examined for a suitable torsion–free connection arising from the Bott connection. The connection has a number of nice geometric properties, however, it is not metric. The metrization leads to a connection with torsion. This metrized connection apprears to be the unique affine connection adapted to each of the foliations such that the endomorphisms of the tangent bundles induced by the partial torsions are both self-conjugate. Investigation of the geometry of the connection leads to a tensor which may be treated as a measure of the "lack of the symmetry" of the Weingarten operator, so it encodes the extrinsic geometry of both foliations. This tensor is also a conformal invariant [3].

[1] Ciska M., *Modulus of pairs of foliations*, PhD thesis, University of Lodz, Poland, 2011.

[2] Ciska M., Pierzchalski A., *On the modulus of level sets of conjugate submersions*, Differential Geometry and its Applications 36 (2014), 90–97.

[3] Kalina J., Kozłowski W., Pierzchalski A., *On the Geometry of a pair of foliations and a conformal invariant*, Tohoku Mathematical Journal, to appear.

[4] Rovenski V., Zawadzki T., *Variations of the total mixed scalar curvature of a distribution.*, Annals of Global Analysis and Geometry 54 (2018), no. 1, 87–122.

Higher order algebroids and representations up to homotopy

Mikołaj Rotkiewicz

University of Warsaw
Faculty of Mathematics, Informatics and Mechanics
email: M.Rotkiewicz@mimuw.edu.pl

Abstract

The concept of a higher algebroid, as introduced by M. Jóźwikowski and M. Rotkiewicz, naturally generalizes the notions of an algebroid and a higher tangent bundle. The idea is based on a description of (Lie) algebroids as differential relations of a special kind. My goal is to explain the notion of a higher algebroid in a more standard language, i.e. in terms of some bracket operations and vector bundle morphisms. In order two we end up with representation up to homotopy of (Lie) algebroids.

New geometric structures on 3-manifolds through generalized geometry

Roberto Rubio

Universitat Autònoma de Barcelona
Department of Mathematics
email: roberto.rubio@uab.cat

Abstract

Generalized geometry has used Dirac structures [3] and Courant algebroids [6] to offer frameworks where classical structures join each other in some overarching new structure. For instance, generalized complex geometry [5, 4] encompass complex and symplectic structures. But there is more to it: symplectic foliations with a holomorphic transverse structure are also described by a generalized complex structure. In fact, although every generalized complex manifold must be almost complex, there are 4-dimensional examples of generalized complex manifolds that admit neither complex nor symplectic structures [2].

Generalized geometry admits various starting setups [1]. For odd-dimensional manifolds, a very convenient one is B_n-generalized geometry, so labelled by the role played by $\mathrm{SO}(2n+1)$, a group of Lie type B_n. By direct analogy with generalized complex geometry, we define B_n-generalized complex structures [7], or simply B_n-structures, which now encompass cosymplectic and normal almost complex structures, odd-dimensional analogues of symplectic and complex structures. But again, there is much more to it.

In this talk I will give a straightforward introduction to generalized complex geometry using, instead of Dirac structures and Courant algebroids, differential forms and an underlying Clifford action. This is also the shortest path to then introduce and grasp B_n-structures. Finally, I will comment on recent joint work with Joan Porti about 3-manifolds, which includes examples of manifolds admitting a B_3-structure but not a cosymplectic or a normal almost contact one.

[1] Baraglia D., *Leibniz algebroids, twistings and exceptional generalized geometry*, Journal of Geometry and Physics 62 (2012), no. 5, 903–934.

[2] Cavalcanti G.R., Gualtieri M., *A surgery for generalized complex structures on 4-manifolds*, Journal of Differential Geometry 76 (2007), no. 1, 35–43.

[3] Courant T.J., *Dirac manifolds*, Transactions of the American Mathematical Society 319 (1990), no. 2, 631–661.

[4] Gualtieri M., *Generalized complex geometry*, Annals of Mathematics 174 (2011), no. 1, 75–123.

[5] Hitchin N., *Generalized Calabi-Yau manifolds*, The Quarterly Journal of Mathematics 54 (2003), no. 3, 281–308.

[6] Liu Z.-J., Weinstein A., Xu P., *Manin triples for Lie bialgebroids*, Journal of Differential Geometry 45 (1997), no. 3, 547–574.

[7] Rubio R., *Generalized Geometry of Type B_n*, Thesis (Ph.D.)–University of Oxford (United Kingdom), ProQuest LLC, Ann Arbor, 2014.

Conformal structures with G_2-twistor distribution

Katja Sagerschnig

Polish Academy of Sciences
Center for Theoretical Physics
email: katja@cft.edu.pl

Abstract

Given a split signature conformal structure on a 4-manifold, there is a naturally defined rank 2 distribution on the circle twistor bundle of selfdual null 2-planes. We will discuss the geometry behind this construction and present a classification of (multiply transtitive) homogeneous 4-dimensional conformal structures for which the symmetry algebra of the rank 2 twistor distribution is the exceptional Lie algebra of type G_2. This is joint work with PawełNurowski (CFT PAS) and Dennis The (UiT). The research leading to the results was supported by the GRIEG project SCREAM.

A geometric journey through the Hamilton-Jacobi theory

Cristina Sardón

Universidad Politécnica de Madrid
email: mariacristina.sardon@upm.es

Abstract

In this talk I will present an exhaustive compendium of geometric Hamilton-Jacobi formulations in different geometric backgrounds.

This talk is not only intended for presenting a Hamilton-Jacobi theory on different manifolds, but also for the introduction and review of different geometric structures with their corresponding use in physical phenomena. For example, contact manifolds are presented as a suitable framework for the formulation of thermodynamics, whilst cosymplectic manifolds are a keypoint in the development of time-dependent dynamics. We will contemplate some particular geometric structures: Poisson and contact manifolds, implicit mechanics, Nambu dynamics, field theory, locally conformal formalisms and some discrete counterparts of the mentioned structures. For such structures, we will develop a Hamilton-Jacobi formulation and we will provide examples of the application on the different corresponding backgrounds.

Perturbations of Fefferman spaces over CR three-manifolds

Arman Taghavi-Chabert

University of Warsaw
Faculty of Physics
email: arman.taghavi-chabert@fuw.edu.pl

Abstract

In 1976, Charles Fefferman constructed, in a canonical way, a Lorentzian conformal structure on a circle bundle over a given strictly pseudoconvex Cauchy-Riemann (CR) manifolds of hypersurface type. It is also known, notably through the work of Sir Roger Penrose and his associates, and that of the Warsaw group led by Andrzej Trautman, that CR three-manifolds underlie Einstein Lorentzian four-manifolds whose Weyl tensors are said to be algebraically special. I will show how these two perspectives are related to each other, by presenting modifications of Fefferman's original construction, where the conformal structure is "perturbed" by some semi-basic one-form, which encodes additional data on the CR three-manifold. Our setup allows us to reinterpret previous works by Lewandowski, Nurowski, Tafel, Hill, and independently, by Mason. Metrics in such a perturbed Fefferman conformal class whose Ricci tensor satisfies certain degeneracy conditions, are only defined off sections of the Fefferman bundle, which may be viewed as "conformal infinity". The prescriptions on the Ricci tensor can then be reduced to differential constraints on the CR three-manifold in terms of a "complex density" and the CR data of the perturbation one-form. One such constraint turns out to arise from a non-linear, or gauged, analogue of a second-order differential operator on densities. A solution thereof provides a criterion for the existence of a CR function and, under certain conditions, for CR embeddability. This talk is based on arxiv:2303.07328.

Totally geodesic submanifolds and homogeneous spheres

Alberto Rodríguez Vázquez

KU Leuven
Department of Mathematics
email: alberto.rodriguezvazquez@kuleuven.be

Abstract

The classification of transitive Lie group actions on spheres was obtained by Borel, Montgomery, and Samelson in the forties. As a consequence of this, it turns out that apart from the round metric there are other Riemannian metrics in spheres that are invariant under the action of a transitive Lie group. These other homogeneous metrics in spheres can be constructed by modifying the metric of the total space of the complex, quaternionic, or octonionic Hopf fibration in the direction of the fibers.

In this talk, I will report on a joint work with Carlos Olmos (Universidad Nacional de Córdoba), where we classified totally geodesic submanifolds in Hopf-Berger spheres. These are those Riemannian homogeneous spheres obtained by rescaling the round metric of the total space of Hopf fibrations by a positive factor in the direction of the fibers.

Hard Lefschetz Property for isometric flows and S^3–actions

Robert Wolak

Uniwersytet Jagielloński
email: robert.wolak@uj.edu.pl

joint work with José Ignacio Royo Prieto
and Martin Saralegi-Aranguren

Abstract

The Hard Lefschetz Property (HLP) is an important property which has been studied in several categories of the symplectic world. For Sasakian manifolds, this duality is satisfied by the basic cohomology (so, it is a transverse property). A new version of the HLP has been recently given in terms of duality of the cohomology of the manifold itself. Both properties were recently proved to be equivalent in the case of K-contact flows. We show that the HLP is naturally defined for the more general category of isometric flows, and for the category of almost-free S^3–actions, which generalizes the rich properties of the 3-Sasakian manifolds. We also show that both versions of the HLP (transversal and global) are equivalent for isometric flows and for certain almost-free S^3–actions.

Variational problems and methods in extrinsic geometry of distributions

Tomasz Zawadzki

University of Łódź
Faculty of Mathematics and Computer Science
email: tomasz.zawadzki@wmii.uni.lodz.pl

Abstract

Extrinsic geometry of a distribution (subbundle of the tangent bundle) on a manifold can be described by second fundamental forms and integrability tensors of that distribution and its orthogonal complement. Using these geometric objects to define functionals of Riemannian metric, we characterize contact metric structures as critical points of one of them [1], examine component of the scalar curvature determined by two non-complementary orthogonal distributions [1], and obtain formulas describing changes of geometry of a foliation along a vector field projectable along its leaves.

[1] Zawadzki T., *A variational characterization of contact metric structures*, Annals of Global Analysis and Geometry 62 (2022), 129–166.

[2] Rovenski V., Zawadzki T., *Variations of the mutual curvature of two orthogonal non-complementary distributions*, arXiv:2210.13116 (2022).