Thematic section

CAFS

Complex Analysis and Function Spaces

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SCHEDULE OF THE SECTION
Complex Analysis and Function Spaces

- Monday – September 4th

  16:00–16:30 Eva A. Gallardo-Gutiérrez, *Insights on the Cesàro operator: shift semigroups and invariant subspaces*

  16:30–17:00 Bartosz Łanucha, *Compressions of the multiplication operator*

  coffee break

- Tuesday – September 5th

  14:30–15:00 Marian Nowak, *Nuclear operators on Banach function spaces*

  15:00–15:30 Pablo Sevilla, *Decoupling inequalities with exponential constants*

  15:30–16:00 Rabia Aktaş Karaman, *Some families of orthogonal polynomials on the cone*

  coffee break

- Thursday – September 7th

  14:00–14:30 Pascal J. Thomas, *Sharp Invertibility in Quotient Algebras of* H^∞

  14:30–15:00 Bartosz Malman, *Removal of singularities of Cauchy integrals*

  15:00–15:30 Radosław Szwedek, *Density of analytic polynomials and R-admissibility of weighted Hardy spaces*

  15:30–16:00 Rafał Czyż, *On the Dirichlet problem for the complex Monge-Ampère operator*

  coffee break

- Friday – September 8th

  14:30–15:00 Frank Kutzschebauch, *Factorization of holomorphic matrices*

  15:00–15:30 Małgorzata Michalska, *De Branges–Rovnyak spaces and local Dirichlet spaces of higher order*

  15:30–16:00 María J. Martín, *On convex harmonic mappings*

On the Dirichlet problem for the complex Monge-Ampère operator

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Abstract

We shall outline briefly the definition and basic properties of the Cegrell classes of plurisubharmonic functions defined on bounded hyperconvex domain in $\mathbb{C}^n$. The aim of this talk is to discuss the existence of the solutions to the Dirichlet problem for the complex Monge-Ampère operator for regular and singular measures. We will recall known results and present some open problems related to this subject.

Insights on the Cesàro operator: shift semigroups and invariant subspaces

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based on a joint work with J. R. Partington

Abstract

Despite the fact that one of the most classical transformations of sequences is the Cesàro operator $\mathcal{C}$, there are still many questions about it unsettled. In the seventies, Kriete and Trutt proved the striking result that the Cesàro operator is *subnormal*, namely, $\mathcal{C}$ has a normal extension. More precisely, if I denotes the identity operator on the classical Hardy space $H^2(\mathbb{D})$, they proved that $I - \mathcal{C}$ is unitarily equivalent to the operator of multiplication by the identity function acting on the closure of analytic polynomials on the space $L^2(\mu, \mathbb{D})$ for a particular measure μ. Nonetheless, it remains unknown a precise description of the closed invariant subspaces of $\mathcal{C}$. In this talk, we will show that a closed subspace M is invariant under $\mathcal{C}$ on $H^2(\mathbb{D})$ if and only if $M^\perp$ is invariant under the C_0-semigroup of composition operators induced by the affine maps $\varphi_t(z) = e^{-t}z + 1 - e^{-t}$ for $t \geqslant 0$ and $z \in \mathbb{D}$. The corresponding result also holds in the Hardy spaces $H^p(\mathbb{D})$ for $1 < p < \infty$. Moreover, in the Hilbert space setting, by linking the invariant subspaces of $\mathcal{C}$ to the lattice of the closed invariant subspaces of the standard right-shift semigroup acting on a particular weighted L^2-space on the line, we will exhibit a large class of non-trivial closed invariant subspaces of $\mathcal{C}$ and provide a complete characterization of the finite codimensional ones. In particular, we will establish the limits of such approach in order to provide a complete description of the lattice of the invariant subspaces of $\mathcal{C}$.

Some families of orthogonal polynomials on the cone

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Abstract

In this work, we present some families of orthogonal polynomials with respect to the weight function $w(t)(t^2 - \|x\|^2)^{\mu-1/2}$, $\mu > -1/2$, on the cone $\{(x,t) : \|x\| \leqslant t, x \in \mathbb{R}^d, t > 0\}$ in $\mathbb{R}^{d+1}$. We define the monomial basis and the basis via Rodrigues formulas on the cone. We give families of orthogonal polynomials by the Rodrigues type formulas when w is the Laguerre weight or the Jacobi weight. Finally, we obtain generating functions for these Rodrigues type bases and show that these families of polynomials are partially biorthogonal.

[1] Aktaş R., Branquinho A., Foulquie-Moreno A., Xu Y., *Monomial and Rodrigues orthogonal polynomials on the cone*, Journal of Mathematical Analysis and Applications 522 (2023), no. 2.

[2] Dunkl C., Xu Y., *Orthogonal polynomials of several variables*, Encyclopedia of Mathematics and its Applications 155, Cambridge Univ. Press, Cambridge, 2014.

[3] Xu Y., *Monomial orthogonal polynomials of several variables*, Journal of Approximation Theory 133 (2005), 1–37.

[4] Xu Y., *Orthogonal polynomials and Fourier orthogonal series on a cone*, Journal of Fourier Analysis and Applications 26 (2020), no. 36.

[5] Xu Y., *Approximation and localized polynomial frame on conic domains*, Journal of Functional Analysis 281 (2021), no. 12.

[6] Xu Y., *Laguerre expansions on conic domains*, Journal of Fourier Analysis and Applications 27 (2021), no. 64.

Factorization of holomorphic matrices

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Abstract

Every complex symplectic matrix in $\mathrm{Sp}_{2n}(\mathbb{C})$ can be factorized as a product of the following types of unipotent matrices (in interchanging order).

(i) : $\begin{pmatrix} I & B \\ 0 & I \end{pmatrix}$, upper triangular with symmetric $B = B^T$.

(ii) : $\begin{pmatrix} I & 0 \\ C & I \end{pmatrix}$, lower triangular with symmetric $C = C^T$.

The optimal number $T(\mathbb{C})$ of such factors that any matrix in $\mathrm{Sp}_{2n}(\mathbb{C})$ can be factored into a product of T factors has recently been established to be 5 by Jin, P. Lin, Z. and Xiao, B.

If the matrices depend continuously or holomorphically on a parameter, equivalently their entries are continuous functions on a topological space or holomorphic functions on a Stein space X, it is by no means clear that such a factorization by continuous/holomorphic unipotent matrices exists. A necessary condition for the existence is the map $X \to \mathrm{Sp}_{2n}(\mathbb{C})$ to be null-homotopic. This problem of existence of a factorization is known as the symplectic Vaserstein problem or Gromov-Vaserstein problem. In this talk we report on the results of the speaker and his collaborators B. Ivarsson, E. Low and of his Ph.D. student J. Schott on the complete solution of this problem, establishing uniform bounds $T(d, n)$ for the number of factors depending on the dimension of the space d and the size n of the matrices. It seems difficult to establish the optimal bounds. However we obtain results for the numbers $T(1, n)$, $T(2, n)$ for all sizes of matrices in joint work with our Ph.D. students G. Huang and J. Schott. Finally we give an application to the problem of writing holomorphic symplectic matrices as product of exponentials.

[1] Doubtsov E., Kutzschebauch F., *Factorization by elementary matrices, null-homotopy and products of exponentials for invertible matrices over rings*, Analysis and Mathematical Physics 9 (2019), no. 3, 1005–1018.

[2] Ivarsson B., Kutzschebauch F., Løw E., *Factorization of symplectic matrices into elementary factors*, Proceedings of the American Mathematical Society 148 (2020), no. 5, 1963–1970.

[3] Schott J., *Holomorphic Factorization of Mappings into* $Sp_{2n}(\mathbb{C})$, arXiv:2207.05389 (2022).

[4] Pengzhan J., Zhangli L., Bo X., *Optimal unit triangular factorization of symplectic matrices*, arXiv:2108.00223 (2021).

Compressions of the multiplication operator

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Abstract

Let M_φ, $\varphi \in L^2(\mathbb{T})$, be the multiplication operator defined on a dense subset of $L^2(\mathbb{T})$ by $f \mapsto \varphi f$ ($f \in L^\infty(\mathbb{T}) \subset L^2(\mathbb{T})$).

In recent years, compressions of multiplication operators are intensely studied. In this talk we are mainly interested in compressions of M_φ to model spaces K_α and their orthogonal complements $L^2(\mathbb{T}) \ominus K_\alpha$, where $K_\alpha = H^2 \ominus \alpha H^2$, α being a nonconstant inner function: $\alpha \in H^\infty$ and $|\alpha| = 1$ a.e. on the unit circle $\mathbb{T}$. That is to say, we focus on properties and various characterizations of truncated Toeplitz operators, dual truncated Toeplitz operators, and their asymmetric versions.

Removal of singularities of Cauchy integrals

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Abstract

Let $\mathcal{C}_g$ denote the Cauchy integral of a function g on the unit circle $\mathbb{T}$:

$$\mathcal{C}_g(z) = \int_{\mathbb{T}} \frac{g(\zeta)}{1 - \bar{\zeta} z} d|\zeta|, \quad z \in \mathbb{D} = \{z : |z| < 1\}.$$

If $\mathcal{C}_g$ happens to be a nice function (say, maybe smooth up to the boundary of the unit disk $\mathbb{D}$, or even more, or somewhat less...) then what can we say about g? Since the operator $g \mapsto \mathcal{C}_g$ has a huge kernel, clearly we won't be able to identify g. It is perhaps quite surprising that we can nevertheless read off non-trivial properties of the support and size of $|g|$ from the partial spectral data $\mathcal{C}_g$. This problem is important in the theory of approximations in the de Branges-Rovnyak spaces, polynomial approximations in the complex plane, and other themes.

In the talk, I will discuss what me and Adem Limani know about this problem, and what results we have established.

On convex harmonic mappings

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Abstract

A *harmonic mapping* is a univalent (one-to-one) complex-valued harmonic function whose real and imaginary parts are not necessarily conjugate. In other words, the Cauchy-Riemann equations need not be satisfied, so the functions need not be analytic.

In this talk, we will mainly focus on *convex* harmonic mappings in the unit disk $\mathbb{D}$, that is, harmonic mappings which map $\mathbb{D}$ onto a convex domain in the complex plane. We will review some of the properties of this family of functions and present some recent results. Other related questions, to be resolved, will be presented as well.

De Branges–Rovnyak spaces and local Dirichlet spaces of higher order

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joint work with B. Łanucha, M. Nowak and A. Sołtysiak

Abstract

We study local Dirichlet spaces of order m introduced by S. Luo, C. Gu and S. Richter in [1] and de Branges–Rovnyak spaces $\mathcal{H}(b)$ generated by nonextreme and rational functions b from the closed unit ball of H^∞. In particular, we give a characterization of functions from local Dirichlet spaces of order m in terms of their m-th derivatives. We also find explicit formulas for b in the case when $\mathcal{H}(b)$ coincides with local Dirichlet space of order m with equality of norms.

[1] Gu S., Luo S., Richter S., *Higher order local Dirichlet integrals and de Branges-Rovnyak spaces*, Advances in Mathematics 385 (2021).

Nuclear operators on Banach function spaces

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Abstract

The concept of nuclear operators between Banach spaces is due to Grothendieck [2], [3]. Nuclear operators are intimately tied to the topological tensor products of Banach spaces. In particular, nuclear operators defined on Banach function spaces have been studied intensively by Swartz [8], Diestel [1], Tong [9], Pietsch [7], Nowak [4], [5], [6]. For Banach spaces X and Y, we present characterizations of nuclear operators: $T : L^\infty(\mu) \to Y$, $T : L^\infty(\mu, X) \to Y$, $T : B(\sum) \to Y$, in terms of their representing vector measures. Moreover, we give formulas for the traces of some kernels operators.

[1] Diestel J., *The Radon-Nikodym property and the coincidence of integral and nuclear operators*, Revue Roumaine de Mathématique Pures et Appliquées 17 (1972), 1611–1620.

[2] Grothendieck A., *Sur les espaces (F) et (DF)*, Summa Brasiliensis Mathematicae 3 (1954), 357–123.

[3] Grothendieck A., *Produits tensoriels topologiques nuclearies*, Memoirs of the American Mathematical Society 16 (1955).

[4] Nowak M., *Nuclear operators and applications to kernel operators*, to appear in Mathematische Nachrichten.

[5] Nowak M., *Nuclear operators on Banach function spaces*, Positivity 25 (2021), no.3, 801–818.

[6] Nowak M., *Nuclear operators on the Banach space of vector-valued essentially bounded measura-ble functions*, Proceedings of the American Mathematical Society 151 (2023), no. 6, 2573–2585.

[7] Pietsch A., *Eigenvalues and s-numbers*, Akadem. Verlagsges, Geest Portig, Leipzig, 1987.

[8] Swartz C., *An operator characterization of vector measures which have Radon-Nikodym derivatives*, Mathematische Annalen 202 (1973), 77–84.

[9] Tong A.E., *Nuclear mappings on $C(X)$*, Mathematische Annalen 194 (1971), 213–224.

Decoupling inequalities with exponential constants

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joint work with Daniel Carando and Felipe Marceca

Abstract

Decoupling inequalities disentangle complex dependence structures of random objects so that they can be analyzed by means of standard tools from the theory of independent random variables. We study decoupling inequalities for vector-valued homogeneous polynomials evaluated at random variables. We focus on providing geometric conditions ensuring decoupling inequalities with good constants depending only exponentially on the degree of the polynomial. Assuming the Banach space has finite cotype we achieve this for classical decoupling inequalities that compare the polynomials with their associated multilinear operators. Under stronger geometric assumptions on the involved Banach spaces, we also obtain decoupling inequalities between random polynomials and fully independent random sums of their coefficients. Finally, we present decoupling inequalities where in the multilinear operator just two independent copies of the random vector are involved.

Density of analytic polynomials and R-admissibility of weighted Hardy spaces

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Abstract

Does the density of analytic polynomials in an H-admissible space is sufficient to the minimality of the space? This question has a purely foundational background, relating fundamental concepts from the theory of H^p spaces.

We show that there is no general relationship between the density of analytic polynomials and the R-admissibility of an H-admissible space. We solve this problem by finding suitable counterexamples of Hardy spaces built upon some weighted Lebesgue spaces.

[1] Sánchez P.E.A., Szwedek R., *Isomorphic copies of ℓ^∞ in the weighted Hardy spaces on the unit disc*, Journal of Fourier Analysis and Applications 29 (2023), no. 3.

Sharp Invertibility in Quotient Algebras of H^∞

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joint work with Artur Nicolau, Myriam Ounaïès

Abstract

Given an inner function $\Theta \in H^\infty(\mathbb{D})$ and $[g]$ in the quotient algebra $H^\infty/\Theta H^\infty$, its quotient norm is $\|[g]\| := \inf\{\|g + \Theta h\|_\infty, h \in H^\infty\}$. We show that when g is normalized so that $\|[g]\| = 1$, the quotient norm of its inverse can be made arbitrarily close to 1 by imposing $|g(z)| \geqslant 1 - \delta$ when $\Theta(z) = 0$ (the only points where one can define unambiguous values for the class $[g]$) if and only if the function Θ satisfies the following property:

$$\lim_{t \to 1} \inf\left\{|\Theta(z)| : z \in \mathbb{D}, \rho(z, \Theta^{-1}\{0\}) \geqslant t\right\} = 1,$$

where ρ is the usual pseudohyperbolic distance in the disc, $\rho(z,w) := \left|\frac{z-w}{1-z\bar{w}}\right|$. This last property may be satisfied or not by an inner function.

When Θ is a Blaschke product, under a condition of "super-separation" of the zeros, this property is equivalent to Θ being a thin Blaschke product.

We show that there exists Blaschke products which are interpolating and fail this property, while some Blaschke products with this property may fail to be interpolating (and thus aren't thin). We exhibit some sufficient conditions, and interesting examples.