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Letters from presidents

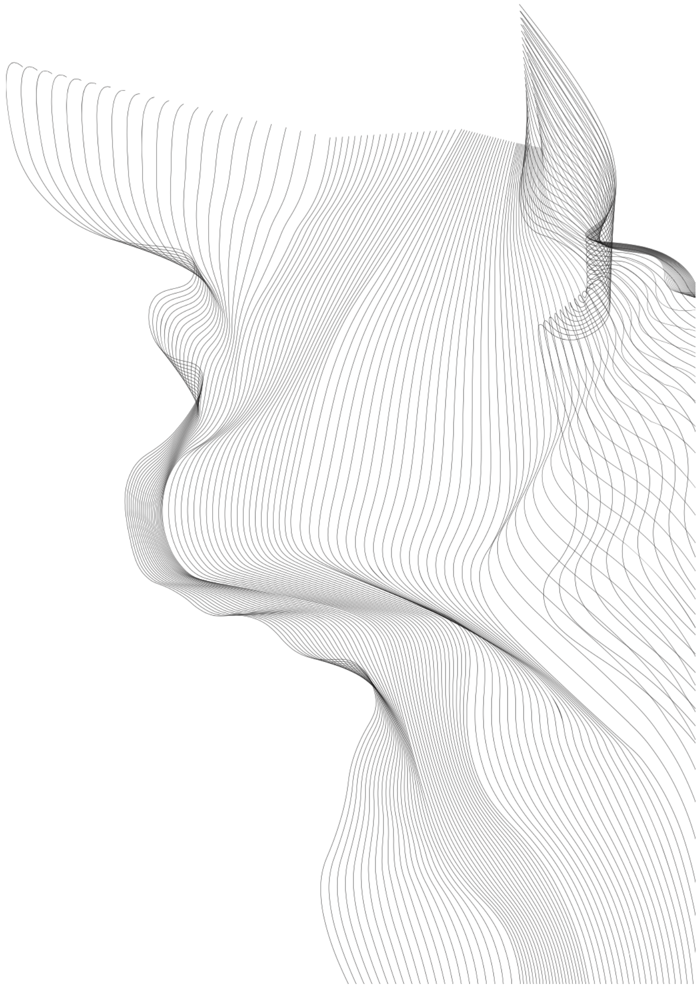
Dear conference attendants of the Spanish-Polish Mathematical Meeting,

As President of the Real Sociedad Matemática Española (RSME), we would like to welcome you to this joint initiative with the mathematical societies Sociedad Española de Matemática Aplicada (SEMA), Societat Catalana de Matemàtiques (SCM) and Polskie Towarzystwo Matematyczne (PTM). In 2019, this initiative emerged in a broad community brought together by the shared pursuit of creating, disseminating, and preserving knowledge in Mathematics in service to the worldwide community. In this regard, the Real Sociedad Matemática Española push the boundaries establishing collaboration with other mathematical societies in order to strive for progress, with a whole-hearted effort and a commitment to high standards. As a scientific society, we do strongly believe in teamwork, creativity, and dedication. Despite the difficulties faced by COVID-19, the conference planned for 2020 will take place in Lodz next September and will highlight some of the latest mathematical findings. I trust you will find them professionally engaging and personally rewarding. Likewise, we hope that this conference represents both mathematical opportunities and benefits for all of you.

Estimados congresistas del Encuentro Matemático Hispano-Polaco,

Como presidenta de la Real Sociedad Matemática Española (RSME), deseo daros la bienvenida a este congreso, fruto de una acción conjunta con las sociedades matemáticas: Sociedad Española de Matemática Aplicada (SEMA), Societat Catalana de Matemàtiques (SCM) y Polskie Towarzystwo Matematyczne (PTM). En 2019, esta iniciativa surgió con el objetivo común de compartir, colaborar, crear y difundir el conocimiento en Matemáticas a nivel internacional. En este sentido, la Real Sociedad Matemática Española ha establecido colaboraciones con otras sociedades con el compromiso por la excelencia. Como sociedad científica, creemos firmemente en la colaboración internacional para potenciar el poder transformador de las Matemáticas que impulsa tanto el conocimiento básico como el desarrollo tecnológico. A pesar de las dificultades surgidas por la pandemia COVID-19, el encuentro que estaba previsto para 2020 tendrá lugar en Lodz el próximo mes de septiembre y contará con conferenciantes destacados en todas las áreas de las Matemáticas. Espero que en esta conferencia cristalice la apuesta por una investigación conjunta y de calidad entre nuestras sociedades, y que a largo plazo pueda ayudar al progreso de la sociedad en beneficio de todos.

Cordialmente,
Eva A. Gallardo Gutiérrez
Presidenta de la Real Sociedad Matemática Española



Manuel J. Castro Díaz
Presidente de la Sociedad Espaõla de Matematica Aplicada

Welcome

A conference is always an opportunity for the research community to share ideas and results, to learn, to listen, and to generate new questions and possible ways to find answers. When the conference is organized, moreover, as a joint meeting of societies, it goes even further. It has the added value of bringing together scientific communities with various specificities, territorial or thematic, in order to work together.

The meeting of the four mathematical societies that we are gathering this 2023 in Lodz, PTM, RSME, SEMA, and SCM, is a good example of this. The tradition of bilateral meetings that the organizing societies have held in recent years with other national societies continues and the collaboration between the four societies is strengthened. Mathematics in general takes centre stage, grouped in thematic sessions. But it is the people who give life to mathematics, excellent researchers from various societies as organizers and speakers, who work together. In particular, collaborations between members of the SCM and the PTM have been going on for years, and meetings like this strengthen them.

We thank the other societies, especially the Polish Mathematical Society, for all their effort and dedication. It has not been easy, because the initial proposal was crushed by the pandemic, but his perseverance has allowed it to be carried out. Organizers, speakers, and participants are making it possible, and we are all invited to work together and share inspiring ideas.

Salutació

Un congrés és sempre una oportunitat per a la comunitat investigadora per compartir idees i resultats, per aprendre, per escoltar-se i per generar noves preguntes i possibles camins per la cerca de respostes. Quan el congrés s'organitza, a més, com una trobada conjunta de societats encara va més enllà. Te el valor afegit d'acostar comunitats científiques amb diverses especificitats, territorials o de temàtica, per tal de treballar conjuntament.

La trobada de les quatre societats matemàtiques que ens apleguem aquest 2023 a Lodz, PTM, RSME, SEMA i SCM, n'és una bona mostra. Continua la tradició de reunions bilaterals que les societats organitzadores han celebrat els darrers anys amb altres societats nacionals, i la col·laboració entre les quatre societats en surt reforçada. El protagonisme és de les matemàtiques en general, agrupades en sessions temàtiques. Però qui dona vida a les matemàtiques son les persones, investigadors excel·lents de les diverses societats com a organitzadors i ponents, que treballen conjuntament. En particular, les col·laboracions entre membres de la SCM i la PTM venen ja de fa anys, i trobades com aquesta les enforteixen.

Agraïm a la resta de societats, però especialment a la Societat Polonesa de Matemàtiques, tot l'esforç i dedicació. No ha estat fàcil, perquè la proposta inicial es va veure estroncada per la pandèmia, però la seva perseverança ha permès dur-lo a terme. Organitzadors, ponents i participants, ho estem fent possible, i tots som convidats a treballar plegats i compartir idees inspiradores.

Montserrat Alsina i Aubach
Presidenta de la Societat Catalana de Matemàtiques

Dear Colleagues,

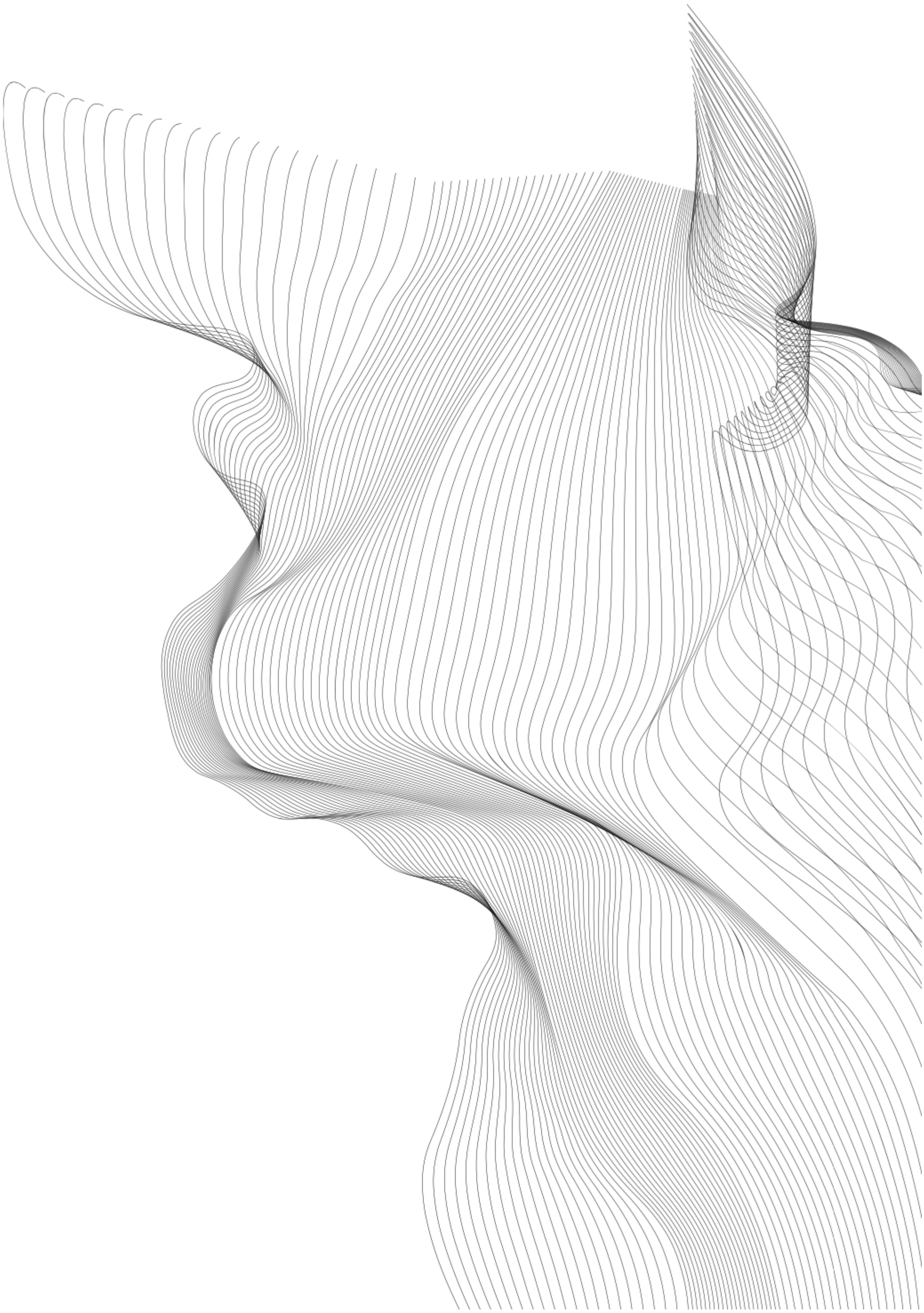
It is a long tradition of Polish Mathematical Society (Polskie Towarzystwo Matematyczne – PTM) to organize joint meetings with other mathematical societies. We had a meeting of the American Mathematical Society and PTM in Warsaw in 2007, Israeli-Polish Mathematical Meeting in Łódź in 2011, joint meeting of Deutsche Mathematiker-Vereinigung and PTM in Poznań in 2014, and a meeting of Unione Matematica Italiana, Società Italiana di Matematica Applicata e Industriale and PTM in Wrocław in 2018.
Now a joint initiative of Real Sociedad Matemática Española, Sociedad Española de Matemática Aplicada, Societat Catalana de Matemàtiques, and PTM finally materializes.
We are happy that we can meet here in Łódź to get know each other, to strengthen our collaborations, to begin new ones. On the behalf of PTM I would like to thank our colleagues from RSME, SEMA, and SCM for help in organization, for patience, when we postponed our meeting due to covid. We have overcome difficulties to be here together.
I wish you all a very fruitful meeting, many scientific discussions but also some time to relax, to enjoy nice social atmosphere, to do some sightseeing, and to return to home places with new ideas and energy to pursue them.

Drogie Koleżanki, Drodzy Koledzy,

Wieloletnią tradycją Polskiego Towarzystwa Matematycznego jest organizowanie wspólnych zjazdów z innymi towarzystwami matematycznymi. Mieliśmy spotkanie American Mathematical Society - PTM w Warszawie w 2007 r., Izraelsko-Polskie Spotkanie Matematyczne w Łodzi w 2011 r., wspólne spotkanie Deutsche Mathematiker-Vereinigung i PTM w Poznaniu w 2014 r. oraz wspólne spotkanie Unione Matematica Italiana, Società Italiana di Matematica Applicata e Industriale i PTM we Wrocławiu w 2018 roku.
Nasza wspólna inicjatywa, Real Sociedad Matemática Española, Sociedad Española de Matemática Aplicada, Societat Catalana de Matemàtiques i PTM wreszcie się materializuje.
Cieszymy się, że możemy się spotkać tutaj w Łodzi, aby się poznać, zacieśnić współpracę, rozpocząć nowe wspólne badania. W imieniu PTM chciałbym podziękować naszym koleżankom i kolegom z RSME, SEMA i SCM za pomoc w organizacji, za cierpliwość, kiedy z powodu covidu przekładaliśmy nasz zjazd. Pokonaliśmy trudności, aby być tutaj razem.
Życzę wszystkim bardzo owocnych spotkań, wielu naukowych dyskusji, ale także czasu na relaks, miłe towarzystwo, zwiedzanie i powrót do rodzinnych stron z nowymi pomysłami i energią do ich realizacji.

Wszystkiego najlepszego,

Jacek Miękisz
Prezes Polskiego Towarzystwa Matematycznego



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- **Leokadia Białas-Cież**, Uniwersytet Jagielloński
- **Małgorzata Bogdan**, Uniwersytet Wrocławski
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- Małgorzata Terepeta, Politechnika Łódzka
- Dariusz Wardowski, Uniwersytet Łódzki
- Zofia Walczak, Uniwersytet Łódzki

LIST OF SECTIONS

APPLICATIONS OF SET THEORY (AST)
Organizers: Antonio Aviles Lopez, Piotr Borodulin-Nadzieja, Szymon Głąb, Jarosław Swaczyna

ARITHMETIC GEOMETRY AND NUMBER THEORY (AGNT)
Organizers: Grzegorz Banaszak, Francesc Bars Cortina, Wojciech Gajda

COMPLEX ANALYSIS AND FUNCTION SPACES (CAFS)
Organizers: Łukasz Kosiński, Manuel Maestre, Daniel Seco, Włodzimierz Zwonek

DIFFERENTIAL GEOMETRY AND GEOMETRIC ANALYSIS (DGGA)
Organizers: Maciej Czarnecki, Miguel Dominguez Vazquez, Jacek Jezierski, Miguel Sanchez Caja

DISCRETE DYNAMICAL SYSTEMS (DDS)
Organizers: Lluis Alseda, Nuria Fagella, Bogusława Karpińska, Anna Zdunik

DYNAMICS OF DIFFERENTIAL EQUATIONS (DDE)
Organizers: Maciej Capiński, Amadeu Delshams, Marcel Guardia, Piotr Zgliczyński

GEOMETRIC AND COMBINATORIAL GROUP THEORY (GCGT)
Organizers: Yago Antolín, Montserrat Casals Ruiz, Damian Osajda, Enric Ventura

GEOMETRY AND TOPOLOGY OF MANIFOLDS (GTM)
Organizers: Vincente Munoz, Krzysztof Pawałowski, Antonio Viruel, Robert Wolak

HILBERT SPACES METHODS (HSM)
Organizers: Dariusz Cichoń, Jose Esteban Gale Gimeno, Eva Gallardo Gutierrez, Paweł Pietrzycki, Jan Stochel, Franciszek Hugo Szafraniec

MATHEMATICAL ECONOMICS (ME)
Organizers: Carlos Herves Beloso, Marta Kornafel, Emma Moreno Garcia, Agnieszka Wiszniewska-Matyszkiel

MATHEMATICS AND STATISTICS IN MACHINE LEARNING (MSML)
Organizers: Enrique Dominguez, Andrzej Nowakowski

MATHEMATICS IN BIOLOGY AND MEDICINE (MBM)
Organizers: Juan Belmonte Beitia, Urszula Foryś

OPERATOR ALGEBRAS (OA)
Organizers: Joan Bosa, Stanisław Goldstein, Bartosz Kwaśniewski, Francesc Perera, Mateusz Wasilewski

ORDERED RANDOM VARIABLES AND RELIABILITY THEORY (ORVR)
Organizers: Jorge Navarro, Tomasz Rychlik

REAL ALGEBRAIC GEOMETRY AND SINGULARITIES (RAGS)
Organizers: Jose Fernando Galvan, Evelia Garcia Barroso, Tadeusz Krasiński, Wojciech Kucharz

MISCELLANEA (M)
Organizers: Andrzej Komisarski, Kamil Niedziałomski

DIDACTICS OF MATHEMATICS (DM) - discussion panel
Moderator: Ryszard Pawlak

SCHEDULE OF THE SPANISH+POLISH MATHEMATICAL MEETING

- Sunday – September 3rd (arrival day)

  16:00–20:00 registration

- Monday – September 4th

  8:00–9:30 registration

  9:30–11:15 opening & PTM prizes

  coffee break

  12:00–13:00 Aleksy Tralle, *Sasakian structures on Smale-Barden manifolds*

  lunch

  14:30–15:00 Borys Kuca, *Arithmetic patterns in large sets*

  15:00–16:00 Guillermo Lorenzo Gomez, *Imaging-informed patient-specific computational modeling of organ-confined prostate cancer*

  16:00–17:00 thematic sections

  coffee break

  17:30–19:00 thematic sections

  19:00–21:00 *reception*

- Tuesday – September 5th

  8:45–9:00 re-opening

  9:00–10:00 Jan Obłój, *Optimal Transport methods for understanding model robustness*

  coffee break

  10:45–11:45 Ana Mancho Sanchez, *A journey from dynamics and geometry to the environment and climate*

  11:45–12:15 Blas Fernández, *Terwilliger algebras beyond distance-regular graphs: a combinatorial approach*

  12:15–12:45 Szymon Cygan, *Stationary solutions to reaction-diffusion-ODE systems*

  lunch

  14:30–16:00 thematic sections

  coffee break

  16:30–18:30 thematic sections

  19:30–21:30 *sport activities/concert*

- Wednesday – September 6th

  9:00–10:00 Joan Bosa Puigredon, *Classification of separable nuclear unital simple C^*-algebras. History and final results*

  10:45–11:45 Piotr Nowak, *Rigidity of groups and algebraic spectral gaps*

  12:00–13:30 discussion panel (DM)/thematic sections

  lunch

  15:00–19:00 *sightseeing*

- Thursday – September 7th

  9:00–10:00 Łukasz Grabowski, *Some applications of expansion in algebra and geometry*

  10:45–11:45 Jesús Álvarez López, *Zeta invariants on Morse forms*

  11:45–12:15 Paweł Walczak, *Geometric flows*

  lunch

  14:00–16:00 thematic sections

  coffee break

  16:30–18:00 thematic sections

  19:30–22:30 *banquet*

- Friday – September 8th

  9:00–10:00 Núria Fagella Rabionet, *Structural stability in complex dynamics*

  coffee break

  10:45–11:45 Andrzej Łuczak, *Entropies of states in von Neumann algebras*

  11:45–12:45 Piotr Zwiernik, *Convex covariance matrix estimation and sparsity*

  10:45–12:45 thematic sections

  lunch

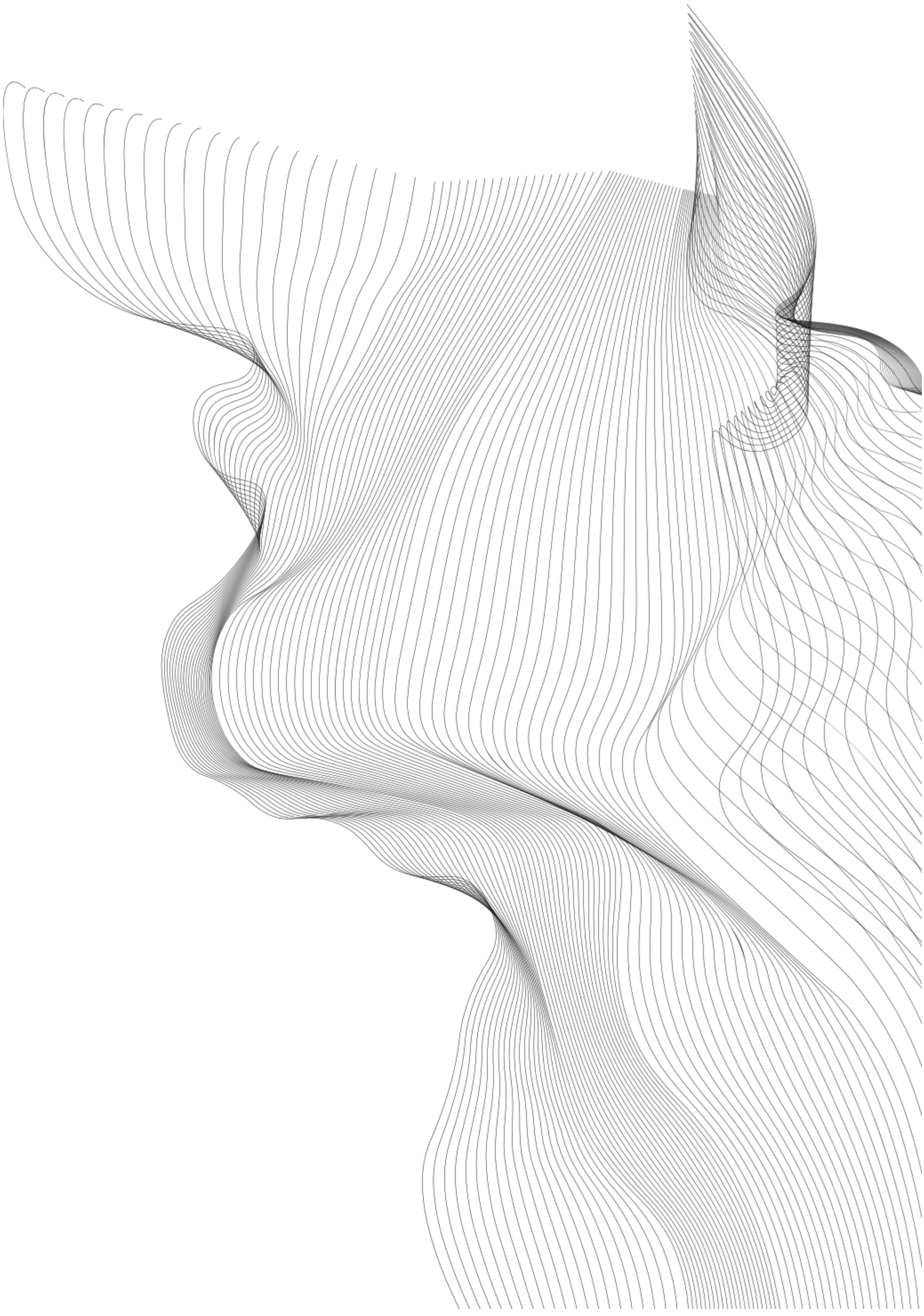
  14:00–15:30 thematic sections

  coffee break

  16:00–17:00 Sławomir Kołodziej, *Solutions of complex Hessian type equations*

  17:00–17:30 closing

- Saturday – September 9th (departure day)



Plenary speakers

- **Jesus Alvarez Lopez**, Universidade de Santiago de Compostela, Spain
- **Joan Bosa Puigredon**, Universitat Autonoma de Barcelona, Spain
- **Nuria Fagella Rabionet**, Universitat de Barcelona, Spain
- **Łukasz Grabowski**, Universitaet Leipzig, Germany
- **Sławomir Kołodziej**, Uniwersytet Jagielloński, Kraków, Poland
- **Guillermo Lorenzo Gomez**, Texas University Austin, USA
- **Andrzej Łuczak**, Uniwersytet Łódzki, Łódź, Poland
- **Ana Mancho Sanchez**, Instituto de Ciencias Matematicas, Madrid, Spain
- **Aleksy Tralle**, Uniwersytet Warmińsko-Mazurski, Olsztyn, Poland
- **Piotr Zwiernik**, University of Toronto, Canada

SCHEDULE OF THE PLENARY LECTURES

- Monday – September 4th

  12:00–13:00 Aleksy Tralle, *Sasakian structures on Smale-Barden manifolds*

  15:00–16:00 Guillermo Lorenzo Gomez, *Imaging-informed patient-specific computational modeling of organ-confined prostate cancer*

- Tuesday – September 5th

  10:45–11:45 Ana Mancho Sanchez, *A journey from dynamics and geometry to the environment and climate*

- Wednesday – September 6th

  9:00–10:00 Joan Bosa Puigredon, *Classification of separable nuclear unital simple C^*-algebras. History and final results*

- Thursday – September 7th

  9:00–10:00 Łukasz Grabowski, *Some applications of expansion in algebra and geometry*

  10:45–11:45 Jesús Álvarez López, *Zeta invariants on Morse forms*

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  9:00–10:00 Núria Fagella Rabionet, *Structural stability in complex dynamics*

  10:45–11:45 Andrzej Łuczak, *Entropies of states in von Neumann algebras*

  11:45–12:45 Piotr Zwiernik, *Convex covariance matrix estimation and sparsity*

  16:00–17:00 Sławomir Kołodziej, *Solutions of complex Hessian type equations*

Zeta invariants of closed 1-forms

Jesús A. Álvarez López

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joint work with Yuri Kordyukov and Eric Leichtnam

Abstract

Let ω be a closed 1-form on a closed Riemannian manifold M. It induces a perturbation of the de Rham complex, $d_z = d + z\,\omega\wedge$, depending on a parameter $z \in \mathbb{C}$. This perturbation was introduced by Witten when ω is exact. The study of its version for a closed 1-form ω was began by Novikov, Pazhitnov and Farber. Taking a Riemannian metric g on M, we also have the corresponding perturbed codifferential and Laplace operators, δ_z and Δ_z. Let Π^1_z denote the orthogonal projection to the image of d_z. The operator $\theta \wedge d_z^{-1}\Pi^1_z$ plays an important role in our study of a trace formula for foliated flows, conjectured by Christopher Deninger. Even though this operator is not of trace class, we need some renormalized version of its super-trace, which should converge to zero as $\Re z \to \infty$.

The talk will be about our efforts and progress to achieve these goals. For $s \in \mathbb{C}$ with $\Re s \gg 0$, the operator $\theta \wedge \delta_z \Delta^{-s}$ is of trace-class, and its super-trace defines a function of s with a meromorphic extension to $\mathbb{C}$; this is the zeta function $\zeta(s, \Delta_z, \theta \wedge \delta_z \mathbf{w})$, where $\mathbf{w}$ denotes the degree involution. Its value at $s = 1$ would be the desired renormalized version of the super-trace of $\theta \wedge d_z^{-1}\Pi^1_z$. However the general theory states that there may a pole at $s = 1$. With appropriate choices of g and ω, defining the same cohomology class, we show that $\zeta(s, \Delta_z, \theta \wedge \delta_z \mathbf{w})$ is regular at $s = 1$, and we have made an important progress to obtain $\zeta(1, \Delta_z, \theta \wedge \delta_z \mathbf{w}) \to 0$ as $\Re z \to \infty$.

Classification of separable nuclear unital simple C^*-algebras. History and final results

Joan Bosa Puigredon

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Abstract

Over the last decade, our understanding of simple, nuclear C^*-algebras has improved a lot. This is thanks to the the interplay between certain topological and algebraic regularity properties, such as nuclear dimension of C^*-algebras, tensorial absorption of suitable strongly self-absorbing C^*-algebras and order completeness of homological invariants. In particular, this is reflected in the Toms-Winter conjecture. In this talk I will speak about this problem, and explain the general classification of nuclear simple C^*-algebras using the finite nuclear dimension (done in two groundbreaking articles by Elliott-Gong-Lin-Niu and Tikuisis-White-Winter). If time permits, I will also show some research built up from the classification just explained.

Structural stability in complex dynamics

Núria Fagella Rabionet

Universitat de Barcelona
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Abstract

Structural stability is a key concept in dynamical systems, which describes those that are stable under perturbations. Equivalently, unstable systems are sytems that experience *bifurcations arbitrarily* nearby In this talk we discuss these concepts in the context of iteration of complex analytic maps, going from the seminal work on families of rational maps of Mañé, Sad, Sullivan in the eighties to some new results and bifurcations of transcendental families.

Some applications of expansion in algebra and geometry

Łukasz Grabowski

Universitaet Leipzig
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joint work with H. Jardon Sanchez and S. Mellick
building on the previous work by T. Hutchcroft and G. Pete

Abstract

This talk is an overview of some results about dynamical systems (here understood as measure preserving actions of finitely generated groups) which use the "spectral gap property" and its variants. The spectral gap property should be intuitively understood as the property of "expanding" on all Borel sets. This property has been instrumental to establish many interesting applications of the theory of measure-preserving group actions. I will focus on some results about measurable equidecompositions of sets in euclidean spaces (joint work with A. Mathe and O. Pikhurkho) and results about cost of equivalence relations with property (T).

Solutions of complex Hessian type equations

Sławomir Kołodziej

Uniwersytet Jagielloński
email: Slawomir.Kolodziej@im.uj.edu.pl

Abstract

It is a survey of recently developed theory of complex m-Hessian equations and their applications in Kähler and Hermitian geometry.

Imaging-informed patient-specific computational modeling of organ-confined prostate cancer

Guillermo Lorenzo Gomez

The University of Texas at Austin, USA
Oden Institute for Computational Engineering and Sciences
and
University of Pavia, Italy
Department of Civil Engineering and Architecture
email: guillermo.lorenzo@utexas.edu

Abstract

The current clinical management of prostate cancer (PCa) enables its detection at early organ-confined stages by combining regular screening and patient classification in risk groups. Although these newly diagnosed tumors are normally organ-confined and do not usually pose a threat to the patient, most PCa cases are prescribed a radical treatment immediately after diagnosis (e.g., surgery or radiotherapy). The limited individualization of the clinical management beyond risk-group definition has led to significant overtreatment and undertreatment rates, which might adversely impact the patients' lives and life expectancy, respectively. Thus, PCa is a paradigmatic disease in which an individualized predictive technology can make a crucial difference in clinical practice, as it would enable to early identify less aggressive tumors that could be safely monitored and potentially lethal tumors that require immediate treatment. To address this critical need, we can use routine clinical and imaging data to construct and parametrize personalized mathematical models of PCa growth including the key mechanisms involved in this pathology. Then, we can run computer simulations with these models to obtain a forecast of the growth of a patient's tumor, which can assist physicians in clinical-decision making. In this talk, I will show that these models can reproduce tumor growth over the local anatomy of a patient's prostate extracted from imaging data, along with the dynamics of the Prostate Specific Antigen (PSA, a ubiquitous biomarker in PCa clinical management). Additionally, I will discuss the importance of the inhibitive effect of growth-induced mechanical stress on PCa and how the compression exerted by concomitant benign prostatic hyperplasia dramatically impedes tumor growth. Finally, I will

argue that these imaging-based models constitute a promising computational technology to assist physicians in providing a personalized clinical management of PCa.

Entropies of states in von Neumann algebras

Andrzej Łuczak

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Faculty of Mathematics and Computer Science
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Abstract

The talk will be devoted to various kinds of entropy of states in a rather general setting. The underlying object will be a von Neumann algebra and the states will be positive normal functionals on this algebra. As an introduction, we shall consider the algebra of all bounded linear operators on a Hilbert space together with the von Neumann entropy defined by means of the canonical trace. Then the Segal entropy, a family of Rényi's entropies and the Tsallis entropy on a semifinite von Neumann algebra will be presented. Next, various notions of relative entropy for two states will be introduced: first the Umegaki entropy on a semifinite von Neumann algebra, and further, general Araki's entropy on an arbitrary von Neumann algebra. As an interesting complement, various quasi-entropies will appear. Finally, we shall discuss the notion of measured entropy in arbitrary as well as semifinite von Neumann algebras.

A journey from dynamics and geometry to the environment and climate

Ana Mancho Sanchez

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Abstract

Finding order in the apparent chaos that seems to govern transport processes in the ocean is a challenge. In this talk I will describe dynamical system tools that are able to highlight beautiful geometries in the ocean and conform an efficient underlying transport network. Two selected cases will be described from this perspective. First, I will explain how this dynamical template accurately described the long-time behavior of the fuel spill subsequent to the sinking of the Oleg Naydenov fishing ship in the Gran Canaria coast, in Spain in April 2015, confirming that these tools were able to identify potentially dangerous regions for these kind of environmental disasters. Additionally, I will also explain the use of these techniques to analyze transport across the Atlantic Meridional Overturning Current (AMOC), a complex convective system in the Atlantic Ocean involved in the distribution of heat, carbon or nutrients, which plays a central role in the Atlantic climate.

Sasakian structures on Smale-Barden manifolds

Aleksy Tralle

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Faculty of Mathematics and Computer Science
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Abstract

Sasakian geometry is an important odd-dimensional counterpart of Kähler geometry. This topic is of great interest for researchers in the fields of differential geometry, algebraic geometry and topology. The seminal book of Boyer and Galicki pointed out several important directions of research and still unsolved problems related to manifolds endowed with K-contact or Sasakian structure. These are the problems of existence and the research program of studying topological properties of such manifolds. In my talk, I will present recent results in this framework in the first difficult case of 5-dimensional simply connected manifolds. These results are obtained together with Vicente Munõz and Matthias Schütt. We systematically develop the theory of quasi-regular Seifert fibrations over orbifolds which are not necessarily smooth and apply it to solve several existence problems. For example, we find all Smale-Barden manifolds admitting null Sasakian structures, and solve several existence problems posed by Boyer and Galicki.

Convex covariance matrix estimation and sparsity

Piotr Zwiernik

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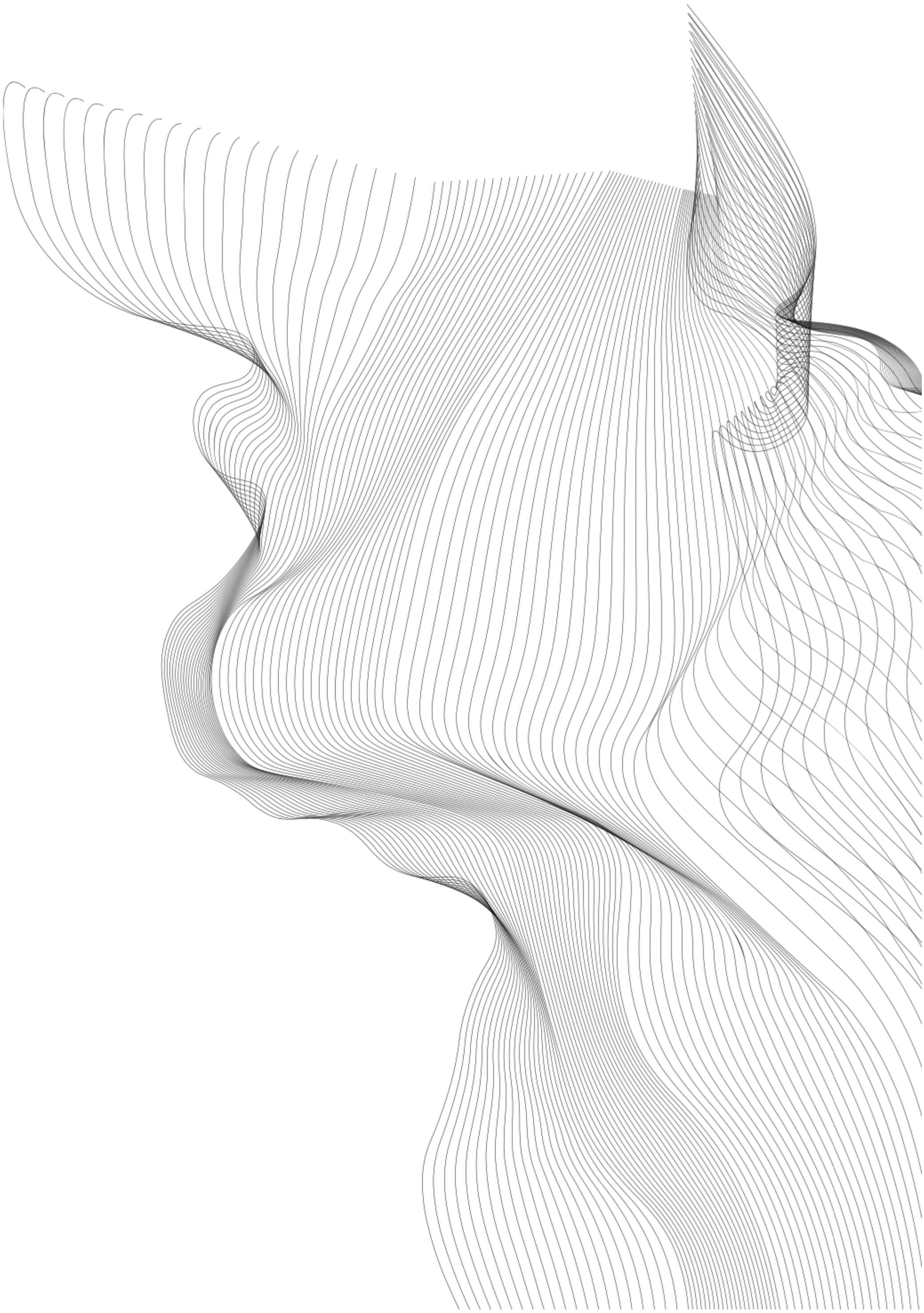
Abstract

In covariance matrix estimation often the challenge is to find a suitable model and an efficient method of estimation. Two popular approaches are to impose linear restrictions on the covariance matrix or on its inverse but linear restrictions on the matrix logarithm of the covariance matrix have been also considered. In this talk I will present a general framework for linear restrictions on various transformations of the covariance matrix. This includes the three examples mentioned above. The proposed estimation method relies on solving a convex problem and leads to an estimator that is asymptotically equivalent to the maximum likelihood estimator of the covariance matrix under the Gaussian assumption. After developing a general theory, we restrict our attention to the case where the linear constraints require certain off-diagonal entries to be zero. Here the geometric picture closely parallels what we know for the Gaussian graphical models.



PTM prize winners

- **Piotr Nowak**, Instytut Matematyczny PAN, Warszawa – Nagroda im. Stefana Banacha
- **Jan Obłój**, Oxford University, UK – Nagroda im. Hugona Steinhausa
- **Marcin E. Kuczma**, Uniwersytet Warszawski – Nagroda im. Samuela Dicksteina
- **Borys Kuca**, University of Crete, Greece – Nagroda dla młodych matematyków
- **Szymon Cygan**, Uniwersytet Wrocławski – International Banach Prize - distinction
- **Blas Fernández**, Univerza na Primorskem, Slovenia – International Banach Prize - distinction

PTM honorary members

- **Stanisław Goldstein**, Uniwersytet Łódzki
- **Paweł Walczak**, Uniwersytet Łódzki

SCHEDULE OF THE PTM PRIZE WINNERS' LECTURES

- Monday – September 4th

  14:30–15:00 Borys Kuca, *Arithmetic patterns in large sets*

- Tuesday – September 5th

  9:00–10:00 Jan Obłój, *Optimal Transport methods for understanding model robustness*

  11:45–12:15 Blas Fernández, *Terwilliger algebras beyond distance-regular graphs: a combinatorial approach*

  12:15–12:45 Szymon Cygan, *Stationary solutions to reaction-diffusion-ODE systems*

- Wednesday – September 6th

  10:45–11:45 Piotr Nowak, *Rigidity of groups and algebraic spectral gaps*

- Thursday – September 7th

  11:45–12:15 Paweł Walczak, *Geometric flows*

Arithmetic patterns in large sets

Borys Kuca

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Abstract

Looking for patterns in sets of numbers is among the oldest and most fundamental mathematical endeavors. A quintessential result in this direction is the Szemerédi theorem which asserts that each subset of integers of positive density contains a finite arithmetic progression of arbitrary length. Often viewed as an example of a "deep" mathematics due to its elaborate and diverse proofs, the Szemerédi theorem has stimulated far-reaching developments in areas as diverse as combinatorics, number theory, harmonic analysis, ergodic theory and model theory. In this talk, I will survey some of the recent progress on the Szemerédi theorem and its generalisations.

Stationary solutions to reaction–diffusion–ODE systems

Szymon Cygan

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joint work with Grzegorz Karch (Wroclaw),
Anna Marciniak-Czochra (Heidelberg) and Kanako Suzuki (Mito)

Abstract

I shall review results on stationary solutions to general reaction-diffusion-ODE systems from Mathematical Biology, where system of ordinary differential equations is coupled with one partial differential equation. Such systems may have different types of stationary solutions, including:

- Sufficiently smooth, regular stationary solutions,
- Jump-discontinuous stationary solutions,
- Singular stationary solutions.

In this talk, I will present the construction of regular and discontinuous stationary solutions and provide sufficient conditions to determine their stability. Furthermore, we will illustrate obtained results in the case of canonical models from Mathematical Biology.

Rigidity of groups and algebraic spectral gaps

Piotr Nowak

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Abstract

The focus of the talk will be Kazhdan's property (T), a powerful rigidity property of groups, and a new approach to proving it via algebraic spectral gaps for the Laplacian in the group ring. This approach grew out of the work of Taka Ozawa and I will in particular present how it was used to prove property (T) for $\mathrm{Aut}\,(F_n)$, the automorphism groups of free groups. I will also discuss some applications and generalizations to higher cohomology and higher index theory.

Optimal Transport methods for understanding model robustness

Jan Obłój

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joint work with many collaborators, including:
D. Bartl, S. Drapeau, S. Eckstein, G. Guo, I. Guo,
B. Joseph, T. Lim, G. Loeper, S. Wang and J. Wiesel

Abstract

Across vast array of applications, mathematics is used to build models which create pathways from inputs to outputs. These models can often be seen as probability measures: discrete (empirical measures over a given data set) or continuous (resulting from an SDE), over finite-dimensional spaces or over pathspaces. The theory of Optimal Transport (OT) offers powerful fully non-parametric tools to measure distances between probability measures, trace geodesics in the space of probability measures, project onto its subsets. In this talk, I will survey some recent advancements that leverage OT tools and intuition, to describe and manage models, helping with selecting/calibrating models and quantifying model uncertainty. I will use questions from mathematical finance as my motivating examples while focusing on providing an overview of the field with its novel mathematical contributions and challenges. In particular, I will discuss robust pricing and hedging and its link to Martingale-OT, non-parametric calibration via Semimartingale-OT, and Wasserstein distributionally robust optimization and the resulting non-parametric Greeks and risk measurements. I will also mention some applications in statistics and machine learning.

Geometric flows

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Abstract

We will survey results on geometric flows of two types:

- *intrinsic* acting on general Riemannian manifolds (see, for instance, [2] and the bibliography therein) like famous *Ricci flow* (see, for example, [3] and the bibliography therein) providing the solution of Poincaré Conjecture by Perelman [6]-[8], and
- *extrinsic* acting on Riemannian manifolds equipped with a foliation or, more generally, arbitary, perhaps nonintergrable, plane fields (see [9] and the bibliorapy therein) and being motivated by the classical *mean curvature flow* (see, among the others, [1, 4, 5] and, again and again, the bibliographies therein).

[1] Brakke K.A., *The motion of the surface by its mean curvature flow*, Princeton Univ. Press, 1978.

[2] Cao H.-D., Yao S.-T., *Geometric Flows*, International Press, 2008.

[3] Chow B., Knopf D., *The Ricci Flow: An Introduction*, American Mathematical Society, 2004.

[4] Huisken G., *Flow by mean curvature of convex surfaces into spheres*, Journal of Differential Geometry 20 (1984), 237 – 266.

[5] Mantegazza C., *Lecture Notes on Mean Curvature Flow*, Birkhäuser, 2011.

[6] Perelman G., *The entropy formula for the Ricci flow and its geometric applications*, arXiv:0211159.

[7] Perelman G., *Ricci flow with surgery on three-manifolds*, arXiv:0303109.

[8] Perelman G., *Finite extinction time for the solutions to the Ricci flow on certain three-manifolds*, arXiv:0307245.

[9] Rovenski V., Walczak P., *Extrinsic Geometry of Foliations*, Birkhäuser, 2021.

Terwilliger algebras beyond distance-regular graphs: a combinatorial approach

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Abstract

The Terwilliger algebra T has been extensively studied in the context of distance-regular graphs, which have only a few irreducible T-modules (up to isomorphism) of a specific endpoint, all of which are thin (with respect to a certain base vertex).

This talk aims to extend these results to irreducible T-modules with endpoint 0 of certain (not necessarily distance-regular) graphs, and shed some new light on their combinatorial properties.

Let Γ be a finite, simple, and connected graph. We examine which vertices x of Γ admit a Terwilliger algebra $T = T(x)$ with an irreducible T-module with endpoint 0, which is thin. We give a purely combinatorial characterization to this algebraic property, which involves the number of certain walks in Γ of a specific shape.

Thematic section

AST
Applications of Set Theory

ORGANIZERS:

Antonio Aviles Lopez (Universidad de Murcia)
Piotr Borodulin-Nadzieja (Uniwersytet Wrocławski)
Szymon Głąb (Politechnika Łódzka)
Jarosław Swaczyna (Politechnika Łódzka)

SCHEDULE OF THE SECTION
Applications of Set Theory

- Monday – September 4th

  16:00–17:00 David Asperó, *Forcing axioms beyond* $H(\omega_2)$

  coffee break

  17:30–18:30 Jorge Lopez-Abad, *On Pelczynski universal space: double Fraïssé classes*

  18:30–19:00 Arturo Martínez-Celis, *Ultraproducts and Michael spaces*

- Tuesday – September 5th

  14:30–15:30 Piotr Koszmider, *Some applications of set theory in Banach spaces*

  15:30–16:00 Alberto Salguero-Alarcón, *Almost disjoint family ties in Banach spaces*

  coffee break

  16:30–17:00 Witold Marciszewski, κ*-Corson compacta and function spaces*

  17:00–18:00 Mikołaj Krupski, κ*-pseudocompactness and uniform homeomorphisms of function spaces*

  18:00–18:30 Piotr Szewczak, *Perfectly meager sets and the Hurewicz property*

- Wednesday – September 6th

  12:00–12:30 Eliza Jabłońska, *Applications of null-finite sets in set-valued map*

  12:30–13:00 Joanna Garbulińska-Węgrzyn, *Erdös-like spaces as Fraïssé limits in some metric categories*

  13:00–13:30 Taras Banakh, *Banakh spaces and their Geometry*

- Thursday – September 7th

  14:00–15:00 Grigor Sargsyan, *Forcing over models of determinacy*

  15:00–15:30 Antonio Avilés, *Examples of compact* $L\Sigma(\leqslant \omega)$*-spaces*

  15:30–16:00 Maciej Korpalski, *Free dimension and isomorphisms of spaces of continuous functions*

  coffee break

  16:30–17:00 Jacek Tryba, *Different kinds of density ideals*

  17:00–17:30 Łukasz Mazurkiewicz, *Ideal analytic sets*

  17:30–18:00 Robert Rałowski, *The Baire theorem, an analogue of the Banach fixed point theorem*

- Friday – September 8th

  14:00–14:30 Szymon Żeberski, *Eggleston meets Mycielski, measure case*

  14:30–15:00 Marcin Michalski, *Eggleston and Mycielski-like theorems for category*

  15:00–15:30 Damian Głodkowski, *A Banach space* $C(K)$ *reading the dimension of* K

  15:30–16:00 Kamil Ryduchowski, *Antiramsey colorings and geometry of Banach spaces*

Forcing axioms beyond $H(\omega_2)$

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Abstract

Forcing axioms assert the existence, given a suitable class Γ of forcing notions, of "sufficiently generic" filters on $\mathcal{P}$, for every $\mathcal{P} \in \Gamma$. In the classical setting, "sufficiently generic" means meeting all members of $\mathcal{D}$, for any family $\mathcal{D}$ of $\aleph_1$-dense subsets of $\mathcal{P}$ fixed in advance. The family of classical forcing axioms, ordered by implication, is known to have a top element, which successfully decides the theory of $H(\omega_2)$. I will survey old and new results concerning the general problem of extending this picture, or finding analogues of it, beyond $H(\omega_3)$. In particular, I will discuss the prospects of finding strong forcing axioms at the level of $H(\omega_3)$ or higher up.

Examples of compact $L\Sigma(\leqslant \omega)$-spaces

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joint work with Mikołaj Krupski

Abstract

The class of $L\Sigma(\leqslant \omega)$-spaces was introduced in 2006 by Kubiś, Okunev and Szeptycki as a natural refinement of the classical and important notion of Lindelöf Σ-spaces. Compact $L\Sigma(\leqslant \omega)$-spaces were considered earlier, under different names, in the works of Tkachuk and Tkachenko in relation to metrizably fibered compacta. In this paper we give counterexamples to several open questions about compact $L\Sigma(\leqslant \omega)$-spaces that are scattered in the literature. Among other things, we refute a conjecture of Kubiś, Okunev and Szeptycki by constructing a separable Rosenthal compactum which is not an $L\Sigma(\leqslant \omega)$-space. We also give insight to the structure of first-countable $(K)L\Sigma(\leqslant \omega)$-compacta.

Support from:
Fundación Séneca - ACyT Región de Murcia project 21955/PI/22, and Agencia Estatal de Investigación and ERDF project PID2021-122126NB-C32.

Banakh spaces and their Geometry

Taras Banakh

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Abstract

In one question posed at `Mathoverflow` I asked whether the real line is a unique metric space (X, d) such that for every positive real number r, every r-sphere $S(c; r) := \{x \in X : d(x, c) = r\}$ in X has cardinality 2 and diameter $2r$. Will Brian proved that this is indeed true in the realm of complete metric spaces. On the other hand, Pietro Mayer suggested an idea of constructing an example showing that without completeness such a characterization of the real line is not true. This motivated studying such metric spaces in more details. To make as much confusion as possible, Will Brian suggested to call a metric space (X, d) *Banakh* if for every point $c \in X$ and real number $r \in d[X^2]$ the sphere $S_d(c; r)$ has cardinality 2 and diameter $2r$. In the talk we shall discuss Banakh spaces and their amusing geometry. In particular, we prove that for every points x, y of a Banakh space X there exists a unique subspace Z of X that contains the points x, y and is isometric to the discrete line $\mathbb{Z}{\cdot}d(x, y)$. Using this geometric fact, we prove that a metric space (X, d) is isometric to a subgroup G of the additive group $\mathbb{Q}$ of rational numbers if and only if X is a Banakh space with $d[X^2] = G_+ := \{x \in G : x \geqslant 0\}$. Generalizing the mentioned result of Will Brian, we prove that a metric space X is isometric to the real line if and only if X is a complete Banakh space such that $G_+ \subseteq d[X^2]$ for some non-cyclic subgroup $G \subseteq \mathbb{Q}$. We prove that every Banakh space (X, d) with $d[X^2] \subseteq \mathbb{Q}$ is isometric to some subgroup of $\mathbb{Q}$. Every Banakh space X satisfies the "rational" axiom of segment construction: for every $x, y \in X$ and every $r \in d[X^2] \cap \mathbb{Q}{\cdot}d(x, y)$ there exists a unique point $z \in X$ such that $d(y, z) = r$ and $d(x, z) = d(x, y) + d(y, z)$. This result implies that for every Banakh space X, the set $d[X^2] \cap \mathbb{Q}$ is a submonoid of the group $\mathbb{Q}$. Yet, for every closed discrete submonoid M of $\mathbb{Q}$ there exists a countable Banakh space (X, d) such that $d[X^2] \cap \mathbb{Q} = M$. Also for every nonzero cardinal $\kappa \leqslant \mathfrak{c}$, the Hilbert space $\ell_2(\kappa)$ contains a discrete subgroup H which is a complete Banakh space of cardinality $|H| = \max\{\kappa, \omega\}$, and a dense $\mathbb{Q}$-linear subspace L such that L is a Banakh space with $d[L^2] = \mathbb{R}_+$.

[1] Banakh T., *Banakh spaces and their Geometry*, arXiv:2305.07354 (2023).

Erdös-like spaces as Fraïssé limits in some metric categories

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Abstract

Considering a category of finite metric spaces with morphisms defined as pairs consisting of isometries and contractions, we get some universal structures as a Fraïssé limit. We investigate connections between obtained objects and Erdös space (or its countable infinite power). Moreover, we prove that Erdös space has Boolean group structures.

A Banach space $C(K)$ reading the dimension of K

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Abstract

We show that (assuming Jensen's diamond principle $\diamondsuit$) for every natural number n there is a Banach space $C(K)$ of continuous functions on a compact Hausdorff space K, such that for every L, if $C(L)$ is isomorphic to $C(K)$ then $\dim L = n$. Constructed spaces are examples of Banach spaces of continuous functions with few operators, which were introduced by Koszmider in [2]. The talk will be based on [1].

[1] Głodkowski D., *A Banach space C(K) reading the dimension of K*, Journal of Functional Analysis 285 (2023), no. 4.

[2] Koszmider P., *Banach spaces of continuous functions with few operators*, Mathematische Annalen 330 (2004), no. 1, 151–183.

Applications of null-finite sets in set-valued map

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Abstract

In [1] the following notion of a "small" set has been introduced: a subset A of a topological group X is called *null-finite* if there exists a sequence $(x_n)_{n\in\omega}$ convergent to 0 in X such that for every $x \in X$ the set $\{n \in \omega : x + x_n \in A\}$ is finite. It turns out that such type of "small" sets can be applied in the theory of set-valued maps.

We present that in some special classes of set-valued maps, i.e.:

- of subadditive set-valued maps,
- of weakly subadditive set-valued maps,
- of midconvex set-valued maps,

upper boundedness on a non-null-finite set implies some kind of regularity of a set-valued map like continuity or local boundedness at every point.

[1] Banakh T., Jabłońska E., *Null-finite sets in topological groups and their applications*, Israel Journal of Mathematics 230 (2019), 361–386.

Free dimension and isomorphisms of spaces of continuous functions

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Abstract

In [1], Marděsić stated a conjecture that whenever a product of d linearly ordered compact spaces can be mapped onto a product of $d + s$ separable infinite compact spaces $K_1, \ldots, K_{d+s}$ with $s \geqslant 1$, then there are at least $s + 1$ metrizable factors K_j . This conjecture was turned into a theorem in [1] using a new notion of free dimension. Such a notion can be used to approach other problems, one of them is a question of isomorphisms between Banach spaces. Surjections between compact spaces are naturally lifting to embeddings of Banach spaces of continuous functions, so free dimension can possibly be applied to spaces of functions defined on e.g. products of compact lines. For products of separable compact lines this question was already solved in [2], but the nonseparable case is an open problem.

[1] Marděsić S., *Mapping products of ordered compacta onto products of more factors*, Glasnik Matematicki Series III 5 (1970), no. 25, 163–170.

[2] Martínez-Cervantes G., Plebanek G., *The Marděsić Conjecture and free products of Boolean algebras.*, Proceedings of the American Mathematical Society 147 (2019), 1763–1772.

[3] Michalak A., *On Banach spaces of continuous functions on finite products of separable compact lines*, Studia Mathematica 251 (2020), 247–275.

Some applications of set theory in Banach spaces

Piotr Koszmider

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Abstract

We will present selected recent applications of set-theoretic methods in Banach spaces.

κ-pseudocompactness and uniform homeomorphisms of function spaces

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Abstract

A Tychonoff space X is called *κ-pseudocompact* if for every continuous mapping f of X into $\mathbb{R}^\kappa$ the image $f(X)$ is compact. This notion generalizes pseudocompactness and gives a stratification of spaces lying between pseudocompact and compact spaces. It is well known that pseudocompactness of X is determined by the uniform structure of the function space $C_p(X)$ of continuous real-valued functions on X endowed with the pointwise topology. Answering a question of Arhangel'skii, we show that analogous assertion is true for κ-pseudocompactness. Our proof relies on fact that κ-pseudocompact can be conveniently characterized by the way X is positioned in its Čech-Stone compactification βX. We shall mention other results concerning the linear-topological structure of the space $C_p(X)$, where this idea can also be applied. The talk is based on the recent paper [1].

[1] Krupski M., *On κ-pseudocompactess and uniform homeomorphisms of function spaces*, Results in Mathematics 78 (2023), no. 154.

On Pelczynski universal space: double Fraïssé classes

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joint work with Jamal K. Kawac
and joint work in progress with S. Todorcevic

Abstract

We present an isomorphic version of the well-known Pelczynski universal space as a Fraïssé limit of what we call a double Fraïssé class. We will discuss the Fraïssé and the KPT correspondence, and the general theory involving other types of structures.

κ-Corson compacta and function spaces

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joint work with Grzegorz Plebanek and Krzysztof Zakrzewski

Abstract

Let κ be an infinite cardinal number. A compact space K is κ-Corson compact if, for some set Γ, K is homeomorphic to a subset of the Σ_κ-product of real lines

$$\Sigma_\kappa(\mathbb{R}^\Gamma) = \{x \in \mathbb{R}^\Gamma : |\{\gamma : x(\gamma) \neq 0\}| < \kappa\}.$$

Obviously, the well known class of Corson compact spaces coincides with the class of ω_1-Corson compact spaces.

We will present some recent results concerning κ-Corson compact spaces and related function spaces.

[1] Marciszewski W., Plebanek G., Zakrzewski K., *Digging into the classes of κ-Corson compact spaces*, arXiv:2107.02513v4.

Ultraproducts and Michael spaces

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Abstract

A Lindelöf space M is a *Michael space* if $M \times \mathbb{N}^{\mathbb{N}}$ is not Lindelöf. The first example of such spaces was constructed by E. Michael [1] using the Continuum Hypothesis and since then, many other consistent examples were constructed. One of them was by J. Moore [2] in which he gave a general framework to construct one of this spaces; the author used this framework to construct a Michael space under $\mathfrak{d} = \mathrm{cov}(\mathcal{M})$. In this talk, we will talk about this framework and we will use the structure of the ultraproduct given by a selective ultrafilter (plus $\varepsilon \geqslant 0$) to construct a Michael space. In particular, we will prove that there is a Michael space after forcing with $\mathcal{P}(\mathbb{N})/\mathrm{Fin}$.

[1] Michael E., *The Product Of A Normal Space And A Metric Space Need Not Be Normal*, Bulletin of the American Mathematical Society 69 (1963), 375–376.

[2] Moore J.T., *Some of the Combinatorics Related to Michael's Problem*, Proceedings of the American Mathematical Society 127 (1999), no. 8, 2459–2467.

Ideal analytic sets

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joint work with Szymon Żeberski

Abstract

For $B \subseteq \omega$ we write

$$FS(B) = \left\{ \sum_{n \in F} n : F \subseteq B \text{ is nonempty and finite} \right\}.$$

A set $A \subseteq \omega$ is called an *IP-set* if there is an infinite $B \subseteq A$ satisfying $FS(B) \subseteq A$. A family of non-*IP-sets* is called *Hindmann ideal*.

Showing analytic completeness of given analytic set A is one of few options to prove, that A is not Borel. In the talk we will discuss results concerning examples of ideals on ω (treated as a subset of Cantor space) constructed in a similar way to Hindmann ideal.

[1] Filipów R., *On Hindman spaces and the Bolzano-Weierstrass property*, Topology and its Applications 160 (2013), no. 15, 2003–2011.

Eggleston and Mycielski-like theorems for category

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joint work with Robert Rałowski and Szymon Żeberski ([2] and [3])

Abstract

Let us recall the following the two following theorems on inscribing special kind rectangles and squares into large subsets of the plane.

Eggleston Theorem [1]
For every conull set $F \subseteq [0,1]^2$ there are a perfect set $P \subseteq [0,1]$ and conull $B \subseteq [0,1]$ such that $P \times B \subseteq F$.

Mycielski Theorem [4]
For every comeager or conull set $X \subseteq [0,1]^2$ there exists a perfect set $P \subseteq [0,1]$ such that $P \times P \subseteq X \cup \Delta$, where $\Delta = \{(x,x) : x \in [0,1]\}$.

We will consider the category variant of the former (comeager instead of conull) in the Cantor space 2^ω and its strengthening via replacing a perfect set with a body of some type of a perfect tree. Mainly we will focus on uniformly perfect trees, Silver trees and Spinas trees. Moreover we will explore the possibility of conjoining the above theorems by demanding that for a comeager set $G \subseteq 2^\omega \times 2^\omega$ there is a comeager set $B \subseteq 2^\omega$ and a tree T of certain kind such that $[T] \times B \subseteq G$ (modulo diagonal) and $[T] \subseteq B$.

[1] Eggleston H. G., *Two measure properties of Cartesian product sets*, The Quarterly Journal of Mathematics 5 (1954), 108–115.

[2] Michalski M., Rałowski R., Żeberski Sz., *Around Eggleston theorem*, arXiv:2307.07020 (2023).

[3] Michalski M., Rałowski R., Żeberski Sz., *Mycielski among trees*, Mathematical Logic Quarterly 67 (2021), 271–281.

[4] Mycielski J., *Algebraic independence and measure*, Fundamenta Mathematicae 61 (1967), 165–169.

The Baire theorem, an analogue of the Banach fixed point theorem

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joint work with M. Morayne

Abstract

We prove that if X is a T_1 second countable compact space, then X is a Baire space if and only if every open subset of X contains a closed subset with nonempty interior. We also prove an analogue of Banach's fixed point theorem for all T_1 compact spaces. Applying the analogue of Banach's fixed point theorem we prove the existence of unique attractors for so called contractive iterated function systems whose Hutchinson operators are closed in compact T_1 spaces.

[1] Morayne M., Rałowski R., *The Baire Theorem, an Analogue of the Banach Fixed Point Theorem and Attractors in Compact Spaces*, Bulletin des Sciences Mathematiques 183 (2023).

Antiramsey colorings and geometry of Banach spaces

Kamil Ryduchowski

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joint work with Piotr Koszmider

Abstract

With every coloring $c\colon [\omega_1]^2 \to \{0,1\}$ we shall associate some nonseparable Banach space X_c. The talk will focus on the following problem: how to translate various combinatorial properties of the coloring c into geometric properties of the space X_c. We will show, among others, that while MA + ¬ CH implies that the geometry of the spaces X_c is quite regular, some interesting phenomena occur when c has some strong antiramsey properties (so strong that the existence of such c is independent of ZFC).

[1] Guzmán O., Hrušák M., Koszmider P., *Almost disjoint families and the geometry of nonseparable spheres*, arXiv:2212.05520 (2023).

[2] Hájek P., Kania T., Russo T., *Separated sets and Auerbach systems in Banach spaces*, Transactions of the American Mathematical Society 373 (2020), 6961–6998.

[3] Koszmider P., Ryduchowski K., *Equilateral and separated sets in some Hilbert generated Banach spaces*, arXiv:2301.07413 (2023).

[4] Koszmider P., Wark H.M., *Large Banach spaces with no infinite equilateral sets*, Bulletin of the London Mathematical Society 54 (2022), 2066–2077.

Almost disjoint family ties in Banach spaces

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Abstract

An almost disjoint family of subsets of $\mathbb{N}$ is just a family $\mathcal{A}$ of infinite subsets of $\mathbb{N}$ such that every two different members of $\mathcal{A}$ have finite intersection. Almost disjoint families produce nice examples of Banach spaces of continuous real-valued functions on compact spaces, or $C(K)$-spaces, for short. Indeed, every almost disjoint family $\mathcal{A}$ produces a compact space $K_{\mathcal{A}}$ –the *Alexandrov-Urysohn* space associated to $\mathcal{A}$– and so we obtain the Banach space $C(K_{\mathcal{A}})$ of continuous functions on $K_{\mathcal{A}}$. However, if we are willing to exploit the machinery of infinite combinatorics to carefully produce peculiar almost disjoint families, then we open door to exotic Banach spaces that serve as counterexamples for both natural and important questions in the theory of $C(K)$-spaces.

We cannot resist to mention one such question: the so-called *complemented subspace problem*, which asked whether every complemented subspace of a $C(K)$-space must be, again, a $C(K)$-space. After more than 50 years, a counterexample was produced in [1] with the invaluable help of almost disjoint families. In doing so, other mysteries about subspaces and quotients of $C(K)$-spaces were also explained. This talk is dedicated to explore several such mysteries, with special attention to the ideas leading to the construction of the counterexample for the complemented subspace problem.

[1] Plebanek G., Salguero-Alarcón S., *The complemented subspace problem for $C(K)$-spaces: A counterexample*, Advances in Mathematics 426 (2023).

Forcing over models of determinacy

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Abstract

We will survey some recent results obtained by forcing over models of determinacy. In one such model, the restriction of the omega-club filter on the first three uncountable cardinals is an ultrafilter in HOD (this answers a question of Ben Neria and Hayut, and is a joint work with Takehiko Gappo), and in another square fails at omega$_3$ (this is a joint work with Larson). The first result is related to Woodin's HOD Conjecture and the second is related to the iterability problem.

Perfectly meager sets and the Hurewicz property

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joint work with Tomasz Weiss and Lyubomyr Zdomskyy

Abstract

We work in the Cantor space with the usual group operation $+$. A set X is *perfectly meager in the transitive sense* if for any perfect set P there is an F_σ-set F containing X such that for every point t the intersection $F \cap (t+P)$ is meager in the relative topology of $t+P$. A set X is *Hurewicz* if for any sequence $\mathcal{U}_0, \mathcal{U}_1, \ldots$ of open covers of X, there are finite families $\mathcal{F}_0 \subseteq \mathcal{U}_0, \mathcal{F}_1 \subseteq \mathcal{U}_1, \ldots$ such that the family $\{\bigcup \mathcal{F}_n : n \in \omega\}$ is a γ-cover of X, i.e., the sets $\{n : x \notin \bigcup \mathcal{F}_n\}$ are finite for all points $x \in X$. Nowik proved that each Hurewicz set which cannot be mapped continuously onto the Cantor set is perfectly meager in the transitive sense. We present results related to the question, whether the same assertion holds for each Hurewicz set with no homeomorphic copy of the Cantor set inside.

The research was funded by the National Science Center, Poland and the Austrian Science Found under the Weave-UNISONO call in the Weave programme, project:
Set-theoretic aspects of topological selections 2021/03/Y/ST1/00122.

Different kinds of density ideals

Jacek Tryba

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Abstract

We consider several classes of ideals described by some densities. We present connections between Erdős-Ulam, density, matrix summability and generalized density ideals, compare these classes of ideals and show that a certain inaccuracy in Farah's definition of density ideals leads to Farah's characterization when density ideals are Erdős-Ulam ideals being incorrect.

We also show that one of these classes of ideals can be used to characterize ideals given by nonpathological submeasures as well as provide solution to the Problem 5 from The Scottish Book.

Eggleston meets Mycielski, measure case

Szymon Żeberski

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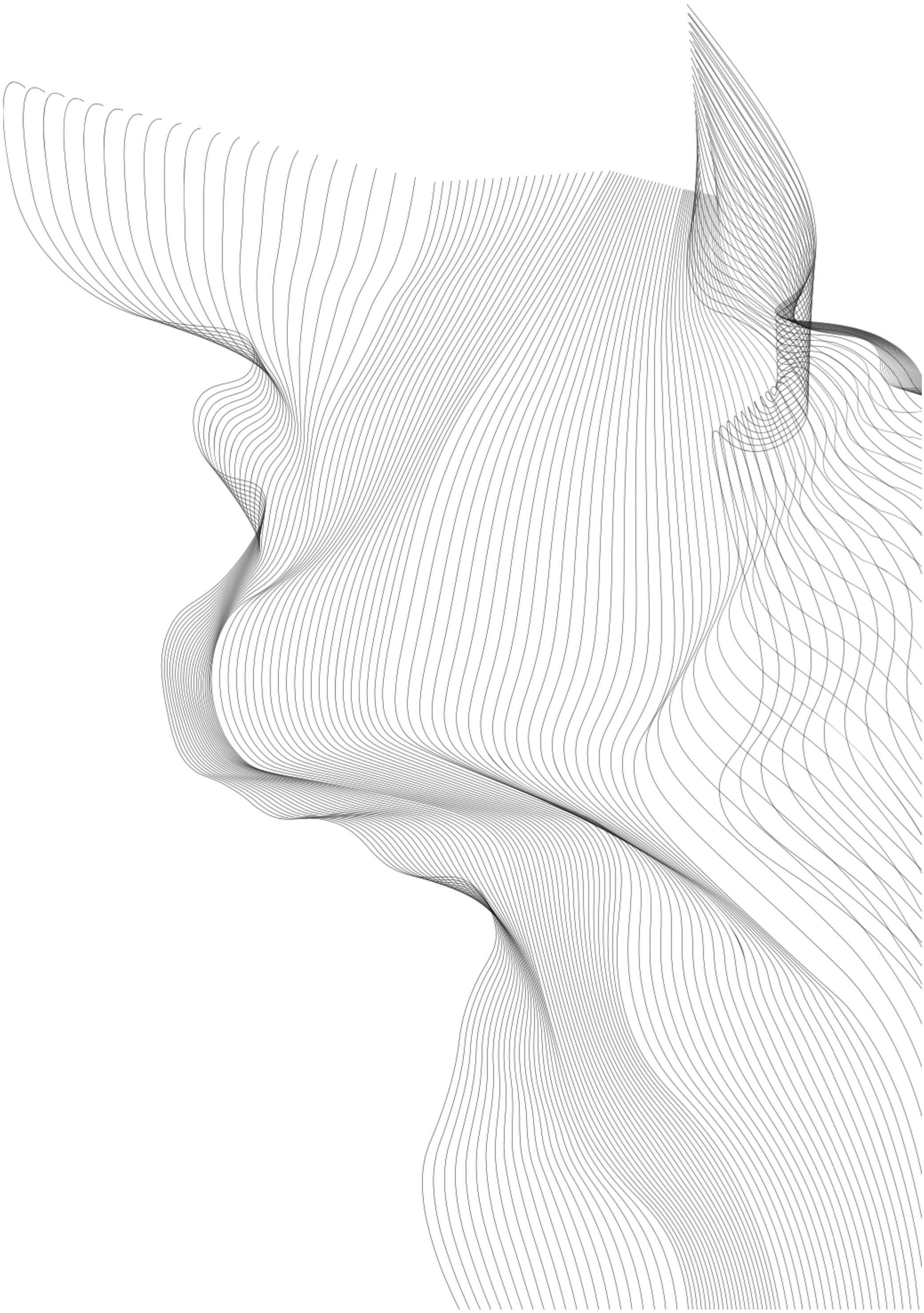
joint work with M. Michalski and R. Rałowski

Abstract

The motivation of this work are the two classical theorems on inscribing rectangles and squares into large subsets of the plane, namely Eggleston Theorem and Mycielski Theorem.

We proved that every conull subset of the plane contains a rectangle $[T] \times H$, where T is a Spinas tree containing a Silver tree and H is conull. Moreover we obtained a common generalization of Eggleston Theorem and Mycielski Theorem stating that every conull subset of the plane contains a rectangle $[T] \times H$ modulo diagonal, where T is a uniformly perfect tree, H is conull and $[T] \subseteq H$.

[1] Michalski M., Rałowski R., Żeberski S., *Around Eggleston theorem*, arXiv: 2307.07020 (2023).



Thematic section

AGNT
Arithmetic Geometry and Number Theory

ORGANIZERS:

Grzegorz Banaszak (Uniwersytet Adama Mickiewicza, Poznań)

Francesc Bars Cortina (Universitat Autonoma de Barcelona)

Wojciech Gajda (Uniwersytet Adama Mickiewicza, Poznań)

SCHEDULE OF THE SECTION
Arithmetic Geometry and Number Theory

- Monday – September 4th

  16:00–16:05 WELCOME

  16:05–17:00 Enric Nart, *Defect in extensions of valuations*

  coffee break

  17:30–18:30 Piotr Achinger, *Maps of affine varieties and wild ramification*

  18:30–19:00 Jędrzej Garnek, *Cohomologies of p-group covers*

- Tuesday – September 5th

  14:30–15:30 Masha Vlasenko, *Frobenius structure and p-adic zeta function*

  15:30–16:00 Álvaro Serrano Holgado, *The generalized Zeta functions of a linear recurrence sequence*

  coffee break

  16:30–17:30 Luis M. Navas, *On a relation between power and Dirichlet series*

  17:30–18:00 Jolanta Marzec-Ballesteros, *Bounds on Fourier coefficients and global sup-norms for Siegel cusp forms of degree* 2

  18:00–18:30 Tomasz Jędrzejak, *Ranks of quadratic twists of Jacobians of generalized Mordell curves*

- Wednesday – September 6th

  12:00–13:00 Zbigniew Hajto, *Real and p-adic Picard-Vessiot extensions*

  13:00–13:30 Teresa Crespo, *Hopf Galois structures*

- Thursday – September 7th

  14:00–15:00 Daniel Macias Castillo, *The refined class number formula for Drinfeld modules*

  15:00–15:30 Bartosz Naskręcki, *Higher moments of families of elliptic curves*

  15:30–16:00 Antonio Rojas-León, *Independence of Gauss sums*

  coffee break

  16:30–17:15 Xavier Guitart, *A quaternionic construction of p-adic singular moduli*

  17:15–18:00 Santiago Molina Blanco, *Waldspurger formulas in higher cohomology*

  18:00 CLOSING

- Friday – September 8th

  14:00–15:00 Joan Carles Lario, *When the Birch–Swinnerton–Dyer conjecture was not remunerated*

Maps of affine varieties and wild ramification

Piotr Achinger

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joint work with Jakob Stix

Abstract

We show that if two nonconstant maps between two affine varieties defined over two possibly different perfect fields induce the same maps on the etale fundamental group, then they are in fact the same up to a power of Frobenius. In fact, it is enough to consider etale cohomology with $\mathbb{F}_p$-coefficients.

When the Birch–Swinnerton–Dyer conjecture was not remunerated

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Abstract

In this talk I will present a personal overview on the chain of events that led to the conjecture of Birch and Swinnerton-Dyer.

Special emphasis will be placed on the role played by Kurt Heegner, through his famous article where Gauss's problem on the determination of imaginary quadratic fields with number of classes one is solved, along with relevant results on the classical problem of congruent numbers.

We will finish with a note on the determination of quadrilateral numbers introduced by Kummer that was proposed to Heegner by his teacher Schwatrz (the son-in-law of Kummer).

Hopf Galois structures

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Abstract

For a Galois extension L/K with Galois group G, the action of G on L by K-automorphisms extends to an action of the group algebra $K[G]$ on L, by endomorphisms of L as a K-vector space. A Hopf Galois structure on a field extension L/K consists in a K-Hopf algebra H together with an action of H on L, satisfying conditions which mimic the properties of the action of $K[G]$ in the Galois case. For a finite separable extension L/K, Greither and Pareigis [3] obtained that Hopf Galois structures may be determined in terms of group theory. In my talk I will introduce Hopf Galois structures on finite separable field extensions and present some recent results on extensions of prime power degree ([1], [1]).

[1] Crespo T., *Automatic realization of Hopf Galois structures*, Journal of Algebra and Its Applications 21 (2022), no. 2.

[2] Crespo T., Salguero M., *Hopf Galois structures on separable field extensions of odd prime power degree*, Journal of Algebra 519 (2019), 424–439.

[3] Greither C., Pareigis B., *Hopf Galois theory for separable field extensions*, Journal of Algebra 106 (1987), 239–258.

Cohomologies of p-group covers

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Abstract

Studying cohomology of a variety with an action of a finite group is a classical and well-researched topic. However, most of the previous results focus either on the tame ramification case, on some special groups, or on specific curves. In the talk, we will consider the case of a curve over a field of characteristic p with an action of a finite p-group. Our research suggests that the Hodge and de Rham cohomologies decompose as sums of certain 'local' and 'global' parts. The global part should be determined by the 'topology' of the cover, while the local parts should depend only on an analytical neighborhood of the fixed points of the action. In fact, the local parts should come from cohomologies of Harbater–Katz–Gabber curves, i.e. covers of the projective line ramified only over ∞. During the talk, we present our results related to this conjecture. As an application, we compute the de Rham cohomologies of $\mathbb{Z}/p$-covers and Klein four covers.

A quaternionic construction of p-adic singular moduli

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Abstract

Rigid meromorphic cocycles were introduced by Henri Darmon and Jan Vonk as a conjectural p-adic extension of singular moduli over real quadratic base fields. They are certain cohomology classes of SL_2 that can be evaluated at real quadratic quantities and the resulting values are conjectured to be algebraic. In this talk I will explain joint work with Marc Masdeu and Xavier Xarles in which we propose a similar construction of cohomology classes in quaternion algebras over totally real fields F. These classes can be evaluated at elements of quadratic extensions K/F, and we conjecture that the resulting values belong to abelian extensions of K. This conjecture is supported by numerical evidence.

Real and p-adic Picard-Vessiot extensions

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Abstract

I will survey Galois theory for partial differential systems defined over formally real differential fields with a real closed field of constants and over formally p-adic differential fields with a p-adically closed field of constants. For an integrable partial differential system defined over such a field, there exists a formally real (resp. formally p-adic) Picard-Vessiot extension. I will comment on the uniqueness result for these Picard-Vessiot extensions and the Galois correspondence theorem in this setting. I will explain the application of this theorem to characterise formally real Liouvillian extensions of real partial differential fields with a real closed field of constants by means of split solvable linear algebraic groups. In this context, I will discuss the topological properties of real Liouville functions and relate them with the concept of tame topology in the sense of Grothendieck and Khovanskii. Finally, I will discuss some possibilities for further development of this theory.

[1] Crespo T., Hajto Z., Mohseni R., *Real Liouvillian Extensions of Partial Differential Fields*, SIGMA 17 (2021), no. 95.

[2] Crespo T., Hajto Z., van der Put M., *Real and p-adic Picard-Vessiot fields*, Mathematische Annalen 365 (2016), 93–103.

Ranks of quadratic twists of Jacobians of generalized Mordell curves

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Abstract

Consider a two-parameter family of hyperelliptic curves $C_{q,b} : y^2 = x^q - b^q$ defined over $\mathbb{Q}$, and their Jacobians $J_{q,b}$ where q is an odd prime and without loss of generality b is a non-zero squarefree integer. The curve $C_{q,b}$ is a quadratic twist by b of $C_{q,1}$ (a generalized Mordell curve of degree q). First, we obtain a few upper bounds for the ranks e.g., if $q \equiv 1 \pmod 4$ and any prime divisor of $2b$ not equal to q is a primitive root modulo q then rank $J_{q,b}(\mathbb{Q}) \leqslant (q-1)/2$. Then we focus on $q = 5$ and get the best possible bound (by 1) or even the exact value of rank (0). In particular, we found infinitely many b with any number of prime factors such that rank $J_{5,b}(\mathbb{Q}) = 0$. We deduce as conclusions the complete list (or the bounds for the number) of rational points on $C_{5,b}$ in such cases. Finally, we found for any given q infinitely many non-isomorphic curves $C_{q,b}$ such that rank $J_{q,b}(\mathbb{Q}) \geqslant 1$.

[1] Jędrzejak T., *Ranks of quadratic twists of Jacobians of generalized Mordell curves*, under review.

[2] Juyal A., Moody D., Roy B., *On ranks of quadratic twists of a Mordell curve*, The Ramanujan Journal 59 (2022), 31–50.

The refined class number formula for Drinfeld modules

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joint work with María Inés de Frutos Fernández
and Daniel Martínez Marqués

Abstract

In 2012, Taelman proved an analogue of the Analytic Class Number Formula, for the Goss L-functions that are associated to Drinfeld modules. He also explicitly stated that 'it should be possible to formulate and prove an equivariant version' of this formula.

We formulate and prove an equivariant, or 'refined', version of Taelman's formula.

As a concrete consequence of our general approach, we also derive explicit consequences for the Galois structure of Taelman class groups.

Bounds on Fourier coefficients and global sup-norms for Siegel cusp forms of degree 2

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Abstract

Siegel modular forms of degree n are a natural generalisation of classical modular forms (of degree 1). They are holomorphic functions invariant under the action of (subgroups of) $\mathrm{Sp}_{2n}(\mathbb{Z})$, possess Fourier expansion and - when cuspidal - are square-integrable. One of the most basic and yet unsolved problems concerns the growth of their Fourier coefficients. It is known as Ramanujan-Petersson conjecture when $n = 1$ and as Resnikoff-Saldaña conjecture when $n \geqslant 2$. Deligne's proof of the first conjecture had a significant impact on many problems in mathematics, cf. [3], including an optimal solution to a sup-norm problem given by Xia in [4]. When $n > 1$ hardly anything is known.

In the talk we focus on the case $n = 2$. We will present a special case of Gan-Gross-Prasad conjecture, proven by Furusawa and Morimoto [2], and explain how it led us to the best current bounds on Fourier coefficients, both unconditionally and under GRH. As a consequence we will derive a global sup-norm bound for Siegel cusp forms of degree 2 invariant by $\mathrm{Sp}_4(\mathbb{Z})$ in terms of their weight and L^2-norm.

[1] Comtat F., Marzec-Ballesteros J., Saha A., *Bounds on Fourier coefficients and global sup-norms for Siegel cusp forms of degree 2*, arXiv:2307.07376 (2023).

[2] Furusawa M., Morimoto K., *On the Gross-Prasad conjecture with its refinement for* $(\mathrm{SO}(5), \mathrm{SO}(2))$ *and the generalized Böcherer conjecture*, arXiv:2205.09503 (2022).

[3] Sarnak P., *Some applications of modular forms*, Cambridge University Press, Cambridge (1990).

[4] Xia H., *On* L^∞ *norms of holomorphic cusp forms*, Journal of Number Theory 124 (207), p. 325–327.

Waldspurger formulas in higher cohomology

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Abstract

The classical Waldspurger formula, which computes periods of quaternionic automorphic forms in maximal torus, has been used in a wide variety of arithmetic applications, such as the Birch and Swinnerton-Dyer conjecture in rank 0 situations. This is why this formula is considered the rank 0 analogue of the celebrated Gross-Zagier formula.

On the other hand, Eichler-Shimura correspondence allows us to interpret this quaternionic automorphic form as a cocycle in higher cohomology spaces of certain arithmetic groups. In this way we can realize the corresponding automorphic representation in the etale cohomology of certain Shimura varieties. In this work we find a formula, analogous to that of Waldspurger, which relates cap-products of this cocycle and fundamental classes associated with maximal torus with special values of Rankin-Selberg L-functions.

Defect in extensions of valuations

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joint work with Josnei A. Novacoski

Abstract

Given a valued field (K,v) and a simple finite extension L/K, there are a finite number of valuations $w_1,\dots,w_r$ on L extending v. To each valuation w_i we may associate natural numbers $e(w_i/v)$, $f(w_i/v)$, $d(w_i/v)$ called the *ramification index*, *inertia degree* and *defect*, respectively. These numbers are linked by the formula:

$$[L\colon K]=\sum_{i=1}^{r}e(w_i/v)f(w_i/v)d(w_i/v).$$

The choice of a generator of L/K determines an onto ring homomorphism $K[x]\twoheadrightarrow L$ leading to a reinterpretation of each valuation w on L as a valuation on $K[x]$:

$$\nu\colon K[x]\longrightarrow\!\!\!\!\!\rightarrow L\xrightarrow{\;w\;}\Gamma\cup\{\infty\},$$

where Γ is the value group of w. The Mac Lane-Vaquié theory expresses ν as the last step of a finite chain of *augmentations* of valuations on $K[x]$:

$$\mu_0\longrightarrow\mu_1\longrightarrow\cdots\longrightarrow\mu_{\ell-1}\longrightarrow\mu_\ell=\nu,$$

starting with a very simple (degree-one) valuation μ_0.

We shall define a natural number $d(\mu_n\to\mu_{n+1})$ associated to each augmentation, so that the defect of the extension w/v can be expressed as:

$$d(w/v)=d(\mu_0\to\mu_1)\cdots d(\mu_{\ell-1}\to\mu_\ell).$$

This result was proved by Vaquié under the assumption that (K,v) is henselian.

Higher moments of families of elliptic curves

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joint work with Matija Kazalicki ([2], [1])

Abstract

In this talk we will discuss some new developments in higher moment sums of 1-parametric families of elliptic curves. These sums have connections to modular forms and algebraic curves. I will sketch a result about the second moment of cubic curves leading to a connection with intermediate Jacobians in threefolds. Next we will discuss proofs of modularity of certain rigid Calabi-Yau threefolds which uses directly higher moments, universal families of elliptic curves and Deligne's results, avoiding completely the standard approach via Faltings-Serre method.

[1] Kazalicki M., Naskręcki B., *Second moments and the bias conjecture for the family of cubic pencils*, arXiv:2012.11306 (2021).

[2] Kazalicki M., Naskręcki B., *Diophantine triples and K3 surfaces*, Journal of Number Theory 236 (2022), 41–70.

On a relation between power and Dirichlet series

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Abstract

Given any convergent Laurent series with finite singular part, $f(z) = \sum_{n=-k}^{\infty} a_n z^n$ $(k \geqslant 0)$, we can construct via a truncated Mellin transform a meromorphic function $F(s)$ on $\mathbb{C}$ whose poles are simple and contained in the finite set $\{1, 2, \ldots, k\}$, with residues $\operatorname{Res}(F; n) = a_{-n}/(n-1)!$ and values $F(-n) = (-1)^n n!\, a_n$ for integer $n \geqslant 0$. Such an F is far from unique, though.

When $f(z) = H(e^{-z})e^{-z}$ with $H(z)$ analytic on the complex open unit disk $\mathbb{D}$, there is a corresponding $F(s)$ which is a complete Mellin transform and constitutes a "natural" choice. If the Taylor series at $z = 0$ of $H(z)$ is $\sum_{n=0}^{\infty} h_n z^n$, the existence of a meromorphic continuation of $H(z)$ to $z = 1$ is reflected in the existence of the meromorphic continuation of the Dirichlet series $D(s) = \sum_{n=1}^{\infty} h_{n-1} n^{-s}$ to $\mathbb{C}$, given by $F(s)$, and having several special properties. We provide many examples, including one which is reminiscent of [6].

This construction also allows us to conclude that certain power series on the unit disk do *not* have a meromorphic continuation to $z = 1$, by checking that the corresponding Dirichlet series does not satisfy one or more of the properties the theorem predicts it should have. This kind of result is weaker but also easier (using purely complex analytic techniques) than those of [2] or [3].

The results and methods presented here are contained in the papers [4, 5]. The overall philosophy is related to the non-rigorous formula known as "Ramanujan's Master Theorem" [1].

[1] Amdeberhan T., Espinosa O., Gonzalez I., Harrison M., Mall V. H., Straub A., *Ramanujan's master theorem*, Ramanujan Journal 29 (2012), 103–120.

[2] Bell J.P., Bruin N., Coons M., *Transcendence of generating functions whose coefficients are multiplicative*, Transactions of the American Mathematical Society 364 (2012), no. 2, 933–959.

[3] Knill O., Lesieutre J., *Analytic continuation of Dirichlet series with almost periodic coefficients*, Complex Analysis and Operator Theory 6 (2012), no. 1, 237–255.

[4] Navas L.M., Ruiz F.J., Varona J.L., *Appell polynomials as values of special functions*, Journal of Mathematical Analysis and Applications 459 (2018), no. 1, 419–436.

[5] Navas L.M., Ruiz F.J., Varona J.L., *A connection between power series and Dirichlet series*, Journal of Mathematical Analysis and Applications 493 (2021).

[6] Serrano H.Á., Navas L.M., *The zeta function of a recurrence sequence of arbitrary degree*, Mediterranean Journal of Mathematics 20 (2023), no. 224.

Independence of Gauss sums

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Abstract

Given a finite field $k/\mathbb{F}_p$ and n r-tuples $\mathbf{a}_1, \ldots, \mathbf{a}_n \in \mathbb{Z}^r$ we consider the question of the simultaneous distribution of the normalized Gauss sums $G(\chi^{\mathbf{a}_1}), \ldots, G(\chi^{\mathbf{a}_n})$ in $(S^1)^n$ as χ ranges on the set of characters of $k^{r\times}$. As a consequence, we show that all non-trivial relations among these sums can be expressed as combinations of Frobenius invariance and the Hasse-Davenport product formula.

[1] Rojas-León A., *Equidistribution and independence of Gauss sums*, arXiv:2207.12439.

The generalized Zeta functions of a linear recurrence sequence

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Abstract

Given a Perron number α with minimal polynomial $P(x)$ and a sequence of positive integers $\{a_n\}$ satisfying the linear recurrence determined by $P(x)$, one can define the Dirichlet series associated to $\{a_n\}$,$\sum a_n^{-s}$. In this talk I will show how this Dirichlet series, whose half-plane of convergence is the region $\{\mathfrak{Re}(s) > 0\}$, has an analytic continuation to a meromorphic function $\varphi(s)$ of the whole plane, determining in the process its pole set and residues. It turns out that the pole set depends only on α, not on the particular sequence $\{a_n\}$ chosen, that α can be recovered from the residue at $s = 0$ of $\varphi(s)$, and that some properties of the zeta function $\varphi(s)$ are related to the Diophantine properties of α.

I will also show how, from this analysis of the function $\varphi(s)$ and using a generalisation of a formula of Ramanujan, similar results about analytic continuation can be proved for the Hurwitz-type and Lerch- type Dirichlet series $\sum(a_n + x)^{-s}$ and $\sum z^n(a_n + x)^{-s}$ associated to the sequence $\{a_n\}$.

[1] Serrano Holgado Á., *The Tribonacci Dirichlet series*, Acta Mathematica Hungarica 170 (2023), p. 102–109.

[2] Navas L.M., Serrano Holgado Á., *The zeta function of a recurrence sequence of arbitrary degree*, Mediterranean Journal of Mthematics 20 (2023).

[3] Navas L.M., Serrano Holgado Á., *The Lerch-type zeta function of a recurrence sequence of arbitrary degree*, arXiv:2303.16602 (2023).

Frobenius structure and p-adic zeta function

Masha Vlasenko

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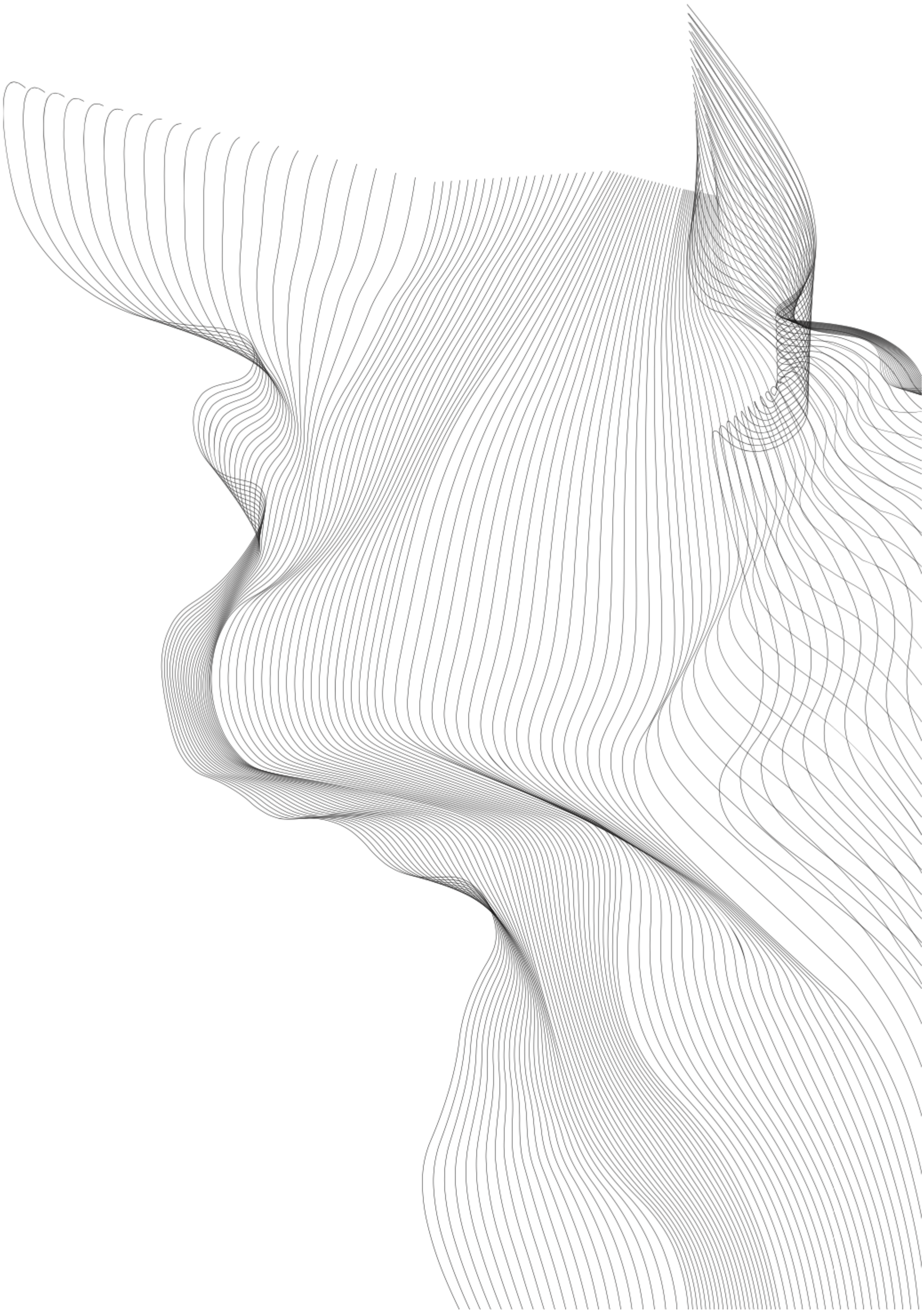
Abstract

I will explain how differential operators coming from algebraic geometry produce interesting p-adic numbers. In a recent work with Frits Beukers we give examples of families of Calabi-Yau hypersurfaces in n dimensions, for which one observes p-adic zeta values $\zeta_p(k)$ for $1 < k < n$. Appearance of p-adic zeta values for differential operators of Calabi-Yau type was conjectured by Candelas, de la Ossa and van Straten.



Thematic section

CAFS

Complex Analysis and Function Spaces

ORGANIZERS:

Łukasz Kosiński (Uniwersytet Jagielloński, Kraków)

Manuel Maestre (Universidad de Valencia)

Daniel Seco (Universidad Carlos III, Madrid)

Włodzimierz Zwonek (Uniwersytet Jagielloński, Kraków)

SCHEDULE OF THE SECTION
Complex Analysis and Function Spaces

- Monday – September 4th

  16:00–16:30 Eva A. Gallardo-Gutiérrez, *Insights on the Cesàro operator: shift semigroups and invariant subspaces*

  16:30–17:00 Bartosz Łanucha, *Compressions of the multiplication operator*

  coffee break

- Tuesday – September 5th

  14:30–15:00 Marian Nowak, *Nuclear operators on Banach function spaces*

  15:00–15:30 Pablo Sevilla, *Decoupling inequalities with exponential constants*

  15:30–16:00 Rabia Aktaş Karaman, *Some families of orthogonal polynomials on the cone*

  coffee break

- Thursday – September 7th

  14:00–14:30 Pascal J. Thomas, *Sharp Invertibility in Quotient Algebras of H^∞*

  14:30–15:00 Bartosz Malman, *Removal of singularities of Cauchy integrals*

  15:00–15:30 Radosław Szwedek, *Density of analytic polynomials and R-admissibility of weighted Hardy spaces*

  15:30–16:00 Rafał Czyż, *On the Dirichlet problem for the complex Monge-Ampère operator*

  coffee break

- Friday – September 8th

  14:30–15:00 Frank Kutzschebauch, *Factorization of holomorphic matrices*

  15:00–15:30 Małgorzata Michalska, *De Branges–Rovnyak spaces and local Dirichlet spaces of higher order*

  15:30–16:00 María J. Martín, *On convex harmonic mappings*

On the Dirichlet problem for the complex Monge-Ampère operator

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Abstract

We shall outline briefly the definition and basic properties of the Cegrell classes of plurisubharmonic functions defined on bounded hyperconvex domain in $\mathbb{C}^n$. The aim of this talk is to discuss the existence of the solutions to the Dirichlet problem for the complex Monge-Ampère operator for regular and singular measures. We will recall known results and present some open problems related to this subject.

Insights on the Cesàro operator: shift semigroups and invariant subspaces

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based on a joint work with J. R. Partington

Abstract

Despite the fact that one of the most classical transformations of sequences is the Cesàro operator $\mathcal{C}$, there are still many questions about it unsettled. In the seventies, Kriete and Trutt proved the striking result that the Cesàro operator is *subnormal*, namely, $\mathcal{C}$ has a normal extension. More precisely, if I denotes the identity operator on the classical Hardy space $H^2(\mathbb{D})$, they proved that $I - \mathcal{C}$ is unitarily equivalent to the operator of multiplication by the identity function acting on the closure of analytic polynomials on the space $L^2(\mu, \mathbb{D})$ for a particular measure μ. Nonetheless, it remains unknown a precise description of the closed invariant subspaces of $\mathcal{C}$. In this talk, we will show that a closed subspace M is invariant under $\mathcal{C}$ on $H^2(\mathbb{D})$ if and only if $M^{\perp}$ is invariant under the C_0-semigroup of composition operators induced by the affine maps $\varphi_t(z) = e^{-t}z + 1 - e^{-t}$ for $t \geqslant 0$ and $z \in \mathbb{D}$. The corresponding result also holds in the Hardy spaces $H^p(\mathbb{D})$ for $1 < p < \infty$. Moreover, in the Hilbert space setting, by linking the invariant subspaces of $\mathcal{C}$ to the lattice of the closed invariant subspaces of the standard right-shift semigroup acting on a particular weighted L^2-space on the line, we will exhibit a large class of non-trivial closed invariant subspaces of $\mathcal{C}$ and provide a complete characterization of the finite codimensional ones. In particular, we will establish the limits of such approach in order to provide a complete description of the lattice of the invariant subspaces of $\mathcal{C}$.

Some families of orthogonal polynomials on the cone

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Abstract

In this work, we present some families of orthogonal polynomials with respect to the weight function $w(t)(t^2 - \|x\|^2)^{\mu-1/2}$, $\mu > -1/2$, on the cone $\{(x,t) : \|x\| \leqslant t,\, x \in \mathbb{R}^d, t > 0\}$ in $\mathbb{R}^{d+1}$. We define the monomial basis and the basis via Rodrigues formulas on the cone. We give families of orthogonal polynomials by the Rodrigues type formulas when w is the Laguerre weight or the Jacobi weight. Finally, we obtain generating functions for these Rodrigues type bases and show that these families of polynomials are partially biorthogonal.

[1] Aktaş R., Branquinho A., Foulquie-Moreno A., Xu Y., *Monomial and Rodrigues orthogonal polynomials on the cone*, Journal of Mathematical Analysis and Applications 522 (2023), no. 2.

[2] Dunkl C., Xu Y., *Orthogonal polynomials of several variables*, Encyclopedia of Mathematics and its Applications 155, Cambridge Univ. Press, Cambridge, 2014.

[3] Xu Y., *Monomial orthogonal polynomials of several variables*, Journal of Approximation Theory 133 (2005), 1–37.

[4] Xu Y., *Orthogonal polynomials and Fourier orthogonal series on a cone*, Journal of Fourier Analysis and Applications 26 (2020), no. 36.

[5] Xu Y., *Approximation and localized polynomial frame on conic domains*, Journal of Functional Analysis 281 (2021), no. 12.

[6] Xu Y., *Laguerre expansions on conic domains*, Journal of Fourier Analysis and Applications 27 (2021), no. 64.

Factorization of holomorphic matrices

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Abstract

Every complex symplectic matrix in $\mathrm{Sp}_{2n}(\mathbb{C})$ can be factorized as a product of the following types of unipotent matrices (in interchanging order).

(i) : $\begin{pmatrix} I & B \\ 0 & I \end{pmatrix}$, upper triangular with symmetric $B = B^T$.

(ii) : $\begin{pmatrix} I & 0 \\ C & I \end{pmatrix}$, lower triangular with symmetric $C = C^T$.

The optimal number $T(\mathbb{C})$ of such factors that any matrix in $\mathrm{Sp}_{2n}(\mathbb{C})$ can be factored into a product of T factors has recently been established to be 5 by Jin, P. Lin, Z. and Xiao, B.

If the matrices depend continuously or holomorphically on a parameter, equivalently their entries are continuous functions on a topological space or holomorphic functions on a Stein space X, it is by no means clear that such a factorization by continuous/holomorphic unipotent matrices exists. A necessary condition for the existence is the map $X \to \mathrm{Sp}_{2n}(\mathbb{C})$ to be null-homotopic. This problem of existence of a factorization is known as the symplectic Vaserstein problem or Gromov-Vaserstein problem. In this talk we report on the results of the speaker and his collaborators B. Ivarsson, E. Low and of his Ph.D. student J. Schott on the complete solution of this problem, establishing uniform bounds $T(d, n)$ for the number of factors depending on the dimension of the space d and the size n of the matrices. It seems difficult to establish the optimal bounds. However we obtain results for the numbers $T(1, n)$, $T(2, n)$ for all sizes of matrices in joint work with our Ph.D. students G. Huang and J. Schott. Finally we give an application to the problem of writing holomorphic symplectic matrices as product of exponentials.

[1] Doubtsov E., Kutzschebauch F., *Factorization by elementary matrices, null-homotopy and products of exponentials for invertible matrices over rings*, Analysis and Mathematical Physics 9 (2019), no. 3, 1005–1018.

[2] Ivarsson B., Kutzschebauch F., Løw E., *Factorization of symplectic matrices into elementary factors*, Proceedings of the American Mathematical Society 148 (2020), no. 5, 1963–1970.

[3] Schott J., *Holomorphic Factorization of Mappings into* $Sp_{2n}(\mathbb{C})$, arXiv:2207.05389 (2022).

[4] Pengzhan J., Zhangli L., Bo X., *Optimal unit triangular factorization of symplectic matrices*, arXiv:2108.00223 (2021).

Compressions of the multiplication operator

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Abstract

Let M_φ, $\varphi \in L^2(\mathbb{T})$, be the multiplication operator defined on a dense subset of $L^2(\mathbb{T})$ by $f \mapsto \varphi f$ $(f \in L^\infty(\mathbb{T}) \subset L^2(\mathbb{T}))$.

In recent years, compressions of multiplication operators are intensely studied. In this talk we are mainly interested in compressions of M_φ to model spaces K_α and their orthogonal complements $L^2(\mathbb{T}) \ominus K_\alpha$, where $K_\alpha = H^2 \ominus \alpha H^2$, α being a nonconstant inner function: $\alpha \in H^\infty$ and $|\alpha| = 1$ a.e. on the unit circle $\mathbb{T}$. That is to say, we focus on properties and various characterizations of truncated Toeplitz operators, dual truncated Toeplitz operators, and their asymmetric versions.

Removal of singularities of Cauchy integrals

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Abstract

Let $\mathcal{C}_g$ denote the Cauchy integral of a function g on the unit circle $\mathbb{T}$:

$$\mathcal{C}_g(z) = \int_{\mathbb{T}} \frac{g(\zeta)}{1-\bar{\zeta}z} d|\zeta|, \quad z \in \mathbb{D} = \{z : |z| < 1\}.$$

If $\mathcal{C}_g$ happens to be a nice function (say, maybe smooth up to the boundary of the unit disk $\mathbb{D}$, or even more, or somewhat less...) then what can we say about g? Since the operator $g \mapsto \mathcal{C}_g$ has a huge kernel, clearly we won't be able to identify g. It is perhaps quite surprising that we can nevertheless read off non-trivial properties of the support and size of $|g|$ from the partial spectral data $\mathcal{C}_g$. This problem is important in the theory of approximations in the de Branges-Rovnyak spaces, polynomial approximations in the complex plane, and other themes.

In the talk, I will discuss what me and Adem Limani know about this problem, and what results we have established.

On convex harmonic mappings

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Abstract

A *harmonic mapping* is a univalent (one-to-one) complex-valued harmonic function whose real and imaginary parts are not necessarily conjugate. In other words, the Cauchy-Riemann equations need not be satisfied, so the functions need not be analytic.

In this talk, we will mainly focus on *convex* harmonic mappings in the unit disk $\mathbb{D}$, that is, harmonic mappings which map $\mathbb{D}$ onto a convex domain in the complex plane. We will review some of the properties of this family of functions and present some recent results. Other related questions, to be resolved, will be presented as well.

De Branges–Rovnyak spaces and local Dirichlet spaces of higher order

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joint work with B. Łanucha, M. Nowak and A. Sołtysiak

Abstract

We study local Dirichlet spaces of order m introduced by S. Luo, C. Gu and S. Richter in [1] and de Branges–Rovnyak spaces $\mathcal{H}(b)$ generated by nonextreme and rational functions b from the closed unit ball of H^∞. In particular, we give a characterization of functions from local Dirichlet spaces of order m in terms of their m-th derivatives. We also find explicit formulas for b in the case when $\mathcal{H}(b)$ coincides with local Dirichlet space of order m with equality of norms.

[1] Gu S., Luo S., Richter S., *Higher order local Dirichlet integrals and de Branges-Rovnyak spaces*, Advances in Mathematics 385 (2021).

Nuclear operators on Banach function spaces

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Abstract

The concept of nuclear operators between Banach spaces is due to Grothendieck [2], [3]. Nuclear operators are intimately tied to the topological tensor products of Banach spaces. In particular, nuclear operators defined on Banach function spaces have been studied intensively by Swartz [8], Diestel [1], Tong [9], Pietsch [7], Nowak [4], [5], [6]. For Banach spaces X and Y, we present characterizations of nuclear operators: $T : L^{\infty}(\mu) \to Y$, $T : L^{\infty}(\mu, X) \to Y$, $T : B(\sum) \to Y$, in terms of their representing vector measures. Moreover, we give formulas for the traces of some kernels operators.

[1] Diestel J., *The Radon-Nikodym property and the coincidence of integral and nuclear operators*, Revue Roumaine de Mathématique Pures et Appliquées 17 (1972), 1611–1620.

[2] Grothendieck A., *Sur les espaces (F) et (DF)*, Summa Brasiliensis Mathematicae 3 (1954), 357–123.

[3] Grothendieck A., *Produits tensoriels topologiques nuclearies*, Memoirs of the American Mathematical Society 16 (1955).

[4] Nowak M., *Nuclear operators and applications to kernel operators*, to appear in Mathematische Nachrichten.

[5] Nowak M., *Nuclear operators on Banach function spaces*, Positivity 25 (2021), no.3, 801–818.

[6] Nowak M., *Nuclear operators on the Banach space of vector-valued essentially bounded measura-ble functions*, Proceedings of the American Mathematical Society 151 (2023), no. 6, 2573–2585.

[7] Pietsch A., *Eigenvalues and s-numbers*, Akadem. Verlagsges, Geest Portig, Leipzig, 1987.

[8] Swartz C., *An operator characterization of vector measures which have Radon-Nikodym derivatives*, Mathematische Annalen 202 (1973), 77–84.

[9] Tong A.E., *Nuclear mappings on $C(X)$*, Mathematische Annalen 194 (1971), 213–224.

Decoupling inequalities with exponential constants

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joint work with Daniel Carando and Felipe Marceca

Abstract

Decoupling inequalities disentangle complex dependence structures of random objects so that they can be analyzed by means of standard tools from the theory of independent random variables. We study decoupling inequalities for vector-valued homogeneous polynomials evaluated at random variables. We focus on providing geometric conditions ensuring decoupling inequalities with good constants depending only exponentially on the degree of the polynomial. Assuming the Banach space has finite cotype we achieve this for classical decoupling inequalities that compare the polynomials with their associated multilinear operators. Under stronger geometric assumptions on the involved Banach spaces, we also obtain decoupling inequalities between random polynomials and fully independent random sums of their coefficients. Finally, we present decoupling inequalities where in the multilinear operator just two independent copies of the random vector are involved.

Density of analytic polynomials and R-admissibility of weighted Hardy spaces

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Abstract

Does the density of analytic polynomials in an H-admissible space is sufficient to the minimality of the space? This question has a purely foundational background, relating fundamental concepts from the theory of H^p spaces.

We show that there is no general relationship between the density of analytic polynomials and the R-admissibility of an H-admissible space. We solve this problem by finding suitable counterexamples of Hardy spaces built upon some weighted Lebesgue spaces.

[1] Sánchez P.E.A., Szwedek R., *Isomorphic copies of ℓ^∞ in the weighted Hardy spaces on the unit disc*, Journal of Fourier Analysis and Applications 29 (2023), no. 3.

Sharp Invertibility in Quotient Algebras of H^∞

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joint work with Artur Nicolau, Myriam Ounaïès

Abstract

Given an inner function $\Theta \in H^\infty(\mathbb{D})$ and $[g]$ in the quotient algebra $H^\infty/\Theta H^\infty$, its quotient norm is $\|[g]\| := \inf\{\|g + \Theta h\|_\infty, h \in H^\infty\}$. We show that when g is normalized so that $\|[g]\| = 1$, the quotient norm of its inverse can be made arbitrarily close to 1 by imposing $|g(z)| \geqslant 1 - \delta$ when $\Theta(z) = 0$ (the only points where one can define unambiguous values for the class $[g]$) if and only if the function Θ satisfies the following property:

$$\lim_{t \to 1} \inf\left\{|\Theta(z)| : z \in \mathbb{D}, \rho(z, \Theta^{-1}\{0\}) \geqslant t\right\} = 1,$$

where ρ is the usual pseudohyperbolic distance in the disc, $\rho(z,w) := \left|\frac{z-w}{1-z\bar{w}}\right|$. This last property may be satisfied or not by an inner function.

When Θ is a Blaschke product, under a condition of "super-separation" of the zeros, this property is equivalent to Θ being a thin Blaschke product.

We show that there exists Blaschke products which are interpolating and fail this property, while some Blaschke products with this property may fail to be interpolating (and thus aren't thin). We exhibit some sufficient conditions, and interesting examples.

Thematic section

DGGA

Differential Geometry and Geometric Analysis

ORGANIZERS:

Maciej Czarnecki (Uniwersytet Łódzki)

Miguel Dominguez Vazquez (Universidade de Santiago de Compostela)

Jacek Jezierski (Uniwersytet Warszawski)

Miguel Sanchez Caja (Universidad de Granada)

SCHEDULE OF THE SECTION
Differential Geometry and Geometric Analysis

- Monday – September 4th

  16:30–17:00 Cristina Sardón, *A geometric journey through the Hamilton-Jacobi theory*

  coffee break

  17:30–18:00 Javier de Lucas, *A characterisation of relative stability in cosymplectic Hamiltonian systems*

  18:00–18:30 Katja Sagerschnig, *Conformal structures with G_2-twistor distribution*

  18:30–19:00 Rouzbeh Mohseni, *Twistors of foliated manifolds*

- Tuesday – September 5th

  14:30–15:00 Mikołaj Rotkiewicz, *Higher order algebroids and representations up to homotopy*

  15:00–15:30 Miguel Ángel Javaloyes, *Finsler spacetimes and its applications to cosmology and wildfire propagation*

  15:30–16:00 Zdeněk Dušek, *Geodesic orbit Finsler (α, β) metrics*

  coffee break

  16:30–17:00 Teresa Arias-Marco, *Mixed Steklov problems on surfaces*

  17:00–17:30 Wojciech Domitrz, *On singularities of the Gauss map components of surfaces in $\mathbb{R}^4$*

  17:30–18:00 Tomasz Zawadzki, *Variational problems and methods in extrinsic geometry of distributions*

- Wednesday – September 6th

  12:00–12:30 Robert Wolak, *Hard Lefschetz Property for isometric flows and S^3–actions*

  12:30–13:00 Antoni Pierzchalski, *Pairs of foliations and a conformal invariant*

  13:00–13:30 José M. Manzano, *On the convergence of minimal graphs and the prescribed mean curvature equation*

- Thursday – September 7th

  14:00–14:30 Piotr Mormul, *From Engel and Cartan to monsters in algebraic and differential geometry*

  14:30–15:00 Kamil Niedziałomski, *An integral formula for G–structures*

  15:00–15:30 Roberto Rubio, *New geometric structures on 3-manifolds through generalized geometry*

  15:30–16:00 Arman Taghavi-Chabert, *Perturbations of Fefferman spaces over CR three-manifolds*

  coffee break

  16:30–17:00 Alberto Rodríguez Vázquez, *Totally geodesic submanifolds and homogeneous spheres*

  17:00–17:30 Daniel Ballesteros Chávez, *On the Isometric embeddings of spheres into de Sitter space*

  17:30–18:00 W. Kryński, *On two constructions in path geometry: dancing and chains*

Mixed Steklov problems on surfaces

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joint work with E. B. Dryden, C. S. Gordon,
A. Hassannezhad, A. Ray, E. Stanhope

Abstract

The talk will focus the attention in the study of eigenvalue asymptotics for mixed-Steklov problems on Riemannian surfaces with Lipschitz boundary of arbitrary genus and arbitrary number of boundary components.

The results obtained in [2] can be viewed as an extension of the result of Girouard, Parnovski, Polterovich and Sher [3] on the full asymptotics of the Steklov spectrum for Riemannian surfaces with smooth boundary, and the result of the authors [1] on the full asymptotics of the Steklov spectrum for orbisurfaces.

[1] Arias-Marco T., Dryden E. B., Gordon C. S., Hassannezhad A., Ray A., Stanhope E., *Spectral geometry of the Steklov problem on orbifolds*, International Mathematics Research Notices 1 (2019), 90–139.

[2] Arias-Marco T., Dryden E. B., Gordon C. S., Hassannezhad A., Ray A., Stanhope E., *Applications of possibly hidden symmetry to Steklov and mixed Steklov problems on surfaces*, arXiv:2301.09010 (2023).

[3] Girouard A., Parnovski L., Polterovich I., Sher D. A., *The Steklov spectrum of surfaces: asymptotics and invariants*, Mathematical Proceedings of Cambridge Philosophical Society 157 (2014), 379–389.

On the Isometric embeddings of spheres into de Sitter space

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Abstract

The Weyl problem is a classical question of the isometric embedding of a positively curved 2-sphere in the Euclidean 3-space. It was considered by Hermann Weyl in 1916, and its resolution by Louis Nirenberg in 1953 led to major advances in the theory of nonlinear differential equations of elliptic type.

In this talk we consider (spacelike) isometric embeddings of a metric on the sphere into de Sitter space, with a suitable curvature restriction. We present an a priori estimate for the mean curvature H of such spacelike hypersurface and some recent developments in these directions.

[1] Ballesteros-Chávez D., Klingenberg W., Lambert B., *Weyl estimates for spacelike hypersurfaces in de Sitter space*, Pacific Journal of Mathematics 320 (2022), no. 1, p. 1–11.

[2] Li C., Wang Z., *The Weyl problem in warped product space*, Journal of Differential Geometry 114 (2020), no. 2, p. 243–304.

On singularities of the Gauss map components of surfaces in $\mathbb{R}^4$

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joint work with L. I. Hernández-Martínez and F. Sánchez-Bringas

Abstract

The Gauss map of a generic immersion of a smooth, oriented surface into $\mathbb{R}^4$ is an immersion. But this map takes values on the Grassmanian of oriented 2-planes in $\mathbb{R}^4$. Since this manifold has a structure of a product of two spheres, the Gauss map has two components that take values on the sphere. We study the singularities of the components of the Gauss map and relate them to the geometric properties of the generic immersion. Moreover, we prove that the singularities are generically stable, and we connect them to the contact type of the surface and $\mathcal{J}$-holomorphic curves with respect to an orthogonal complex structure $\mathcal{J}$ on $\mathbb{R}^4$. Finally, we get some formulas of Gauss-Bonnet type involving the geometry of the singularities of the components with the geometry and topology of the surface.

Geodesic orbit Finsler (α, β) metrics

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part of this work is a joint work with Teresa Arias-Marco

Abstract

Geodesic lemma for homogenenous Finsler (α, β) metrics F is formulated in terms of the underlying Riemannian metric α and the one-form β. The existence of a particular reductive decomposition is described for easy construction of Finslerian geodesic graph, in a suitable group extension. As a consequence, it is proved that for underlying geodesic orbit Riemannian metric α, all Finsler (α, β) metrics F are also geodesic orbit metrics. An alternative construction of Finslerian geodesic graph for naturally reductive underlying Riemannian metric α is also described. The relation of Riemannian geodesic graphs with Finslerian geodesic graphs is illustrated with explicit constructions on spheres. As a corollary, geodesic orbit Finsler (α, β) metrics F on spheres are determined.

[1] Arias-Marco T., Dušek Z., *Geodesic graphs for geodesic orbit Finsler (α, β) metrics on spheres*, arXiv:2303.09368 (2023).

[2] Dušek Z., *Geodesic orbit Finsler (α, β) metrics*, European Journal of Mathematics 9 (2023), no. 9.

Finsler spacetimes and its applications to cosmology and wildfire propagation

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Abstract

We will first show how Finsler spacetimes naturally appear as a tool to solve the time-dependent Zermelo problem in a manifold M, or more generally, the problem of finding the shortest trajectory in time when the velocity is prescribed at any direction and any instant of time, namely, the velocity is a function of the direction and the time. It turns out that the shortest trajectories are the projections to M of lightlike geodesics in the non-relativistic spacetime $\mathbb{R} \times M$, where the first coordinate is the absolute time. These findings can be applied to wildfire propagation models as the velocity of the fire in every direction and instant of time is prescribed, namely, it depends on the wind, the slope, the vegetation, humidity... so the propagation of the fire can be obtained computing the orthogonal lightlike geodesics to the firefront. On the other hand, Finsler spacetimes can be used as cosmological models in situations with a certain degree of anisotropy. We will discuss the meaning of the stress-energy tensor in this context and some proposals for Einstein field equations.

[1] Javaloyes M.A., Pendás-Recondo E., Sánchez M., *Applications of cone structures to the anisotropic rheonomic Huygens' principle*, Nonlinear Analysis 209 (2021).

[2] Javaloyes M.A., Pendás-Recondo E., Sánchez M., *A general model for wildfire propagation with wind and slope*, SIAM Journal of Applied Algebra and Geometry 7 (2023), no. 2, 414–439.

[3] Javaloyes M.A., Sánchez M., *On the definition and examples of cones and Finsler spacetimes.* RACSAM 114 (2020), no. 30.

[4] Javaloyes M.A., Sánchez M., Villaseñor F.F., *On the significance of the Stress-Energy tensor in Finsler spacetimes*, Universe, 8 (2022), no. 2.

[5] Javaloyes M.A., Sánchez M., Villaseñor F.F., *The Einstein-Hilbert-Palatini formalism in pseudo-Finsler geometry*, arXiv:2108.03197 (2021), to appear in Advances in Theoretical and Mathematical Physics.

A characterisation of relative stability in cosymplectic Hamiltonian systems

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Abstract

In short, a relative equilibrium point of a differential equation in a manifold is a point of the manifold whose evolution is described by a symmetry of the differential equation. This talk is concerned with the stability of relative equilibrium points of time-dependent Hamilton equations in symplectic manifolds. More specifically, we aim to analyse the behaviour of trajectories close to solutions evolving via symmetries of time-dependent Hamilton equations. Since relative equilibrium points are not, in general, equilibrium points, standard stability techniques are not available for their study. Other classical techniques to study relative equilibrium points of Hamiltonian systems, like the energy-momentum methods [2], are not enough either since they only cope with time-independent Hamiltonian systems on symplectic manifolds. To generalise the energy-momentum method to a time-dependent realm, we will give a geometric approach to time-dependent Hamiltonian systems through the so-called cosymplectic structures. Next, a new generalisation of the energy-momentum method via cosymplectic geometry will be devised [1]. As an application, the study of restricted circular three-body problems, Lagrange points, and other related physical problems will be accomplished. A special type of cosymplectic-symplectic reduction will be developed and applied.

[1] de Lucas J., Maskalaniec A., Zawora B.M., *A cosymplectic energy-momentum method with applications*, arXiv:2302.05827 (2023).

[2] Marsden J.E., Simo J.C., *The energy momentum method*, Acta Academiae Scientiarum Taurinensis 1 (1988), 245–268.

On the convergence of minimal graphs and the prescribed mean curvature equation

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Abstract

We will see that the prescribed mean curvature equation for spacelike graphs in Lorentz–Minkowski space $\mathbb{L}^3$ can be transformed into the minimal surface equation in $\mathbb{R}^3$ endowed with a certain Riemannian metric with an unit Killing vector field. We will introduce the theory of divergence lines to see that we can produce an entire minimal graph in the Riemannian setting as a limit of graphs over disks. This gives entire spacelike graphs in $\mathbb{L}^3$ with prescribed mean curvature $H \in \mathcal{C}^\infty(\mathbb{R}^2)$ provided that H and ∇H are bounded. This approach also allows us to prescribe the normal of the entire spacelike graph at one point.

[1] Del Prete A., Lee H., Manzano J.M., *A duality for prescribed mean curvature graphs in Riemannian and Lorentzian Killing submersions*, preprint (2023).

[2] Del Prete A., Manzano J.M., Nelli B., *The Jenkins–Serrin problem in 3-manifolds with a Killing vecto field*, preprint (2023).

Twistors of foliated manifolds

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Abstract

Let M^{2n} be an even-dimensional Riemannian manifold, the twistor space $Z(M)$ is the parametrizing space for compatible almost complex structures on M. It is a bundle over M, with fiber $SO(2n)/U(n)$ and is equipped with two almost complex structures $J^{\pm}$, where J^{+} can be integrable but J^{-} is never integrable, however, it still is important as will be discussed.

This talk is based on joint work with R. A. Wolak [1], in which, the theory of twistors on foliated manifolds is developed and some of the works that we did afterward. We construct the twistor space of the normal bundle of a foliation. It is demonstrated that the classical constructions of the twistor theory lead to foliated objects and permit to formulate and prove foliated versions of some well-known results on holomorphic mappings. Since any orbifold can be understood as the leaf space of a suitably defined Riemannian foliation we obtain orbifold versions of the classical results as a simple consequence of the results on foliated mappings.

[1] Mohseni R., Wolak R.A., *Twistor spaces on foliated manifolds*, International Journal of Mathematics 32 (2021), no. 8.

From Engel and Cartan to monsters in algebraic and differential geometry

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Abstract

The aim of the proposed talk is: **firstly** to give an overview of a long tree (not a linear sequence!) of works on the local geometry of flag-like and multi-flag-like distributions in the tangent bundles to manifolds. From classical works of Engel [2], von Weber [11] and E. Cartan, via [a non-exhaustive list:] Semple, Gaspar [3], Kumpera-Rubin [4], Lejeune-Jalabert, Laumond-Risler-Jean, Adachi, Yamaguchi, earlier works of the author, Montgomery-Zhitomirskii [5], Kennedy et al [1], Montgomery-Zhitomirskii [5], till the actual investigations still in progress like [9] and [10].

Secondly to briefly discuss kinematical visualisations of the mentioned objects (so-called car + trailers systems in 2D and articulated arms systems, or spececraft + satellites systems in 3D).

Thirdly to focus on still open issues in the local classification of singularities of 1-flags (i.e., Goursat flags) and of special multi-flags (sometimes also called ‘Goursat multi-flags’).

And, time permitting, **fourthly**, to briefly address two global aspects of the discussed structures: the so-called *Engel geometry* and Mormul’s 2016 conjecture about Goursat structures on closed 5-manifolds.

Why [special] flags? A kind of an answer can be found in the pioneering von Weber’s 1898 work [11]. He started that work with an ambitious plan to go far beyond the classical Frobenius theorem and to locally describe *all* geometric distributions. And he eventually settled for the condition now known as ‘Goursat condition’ (i.e., he settled for the 1-flags).

As for special multi-flags, they started to be extensively dealt with by Kumpera-Rubin around the year 2000, [4]. The – still actual – state of the art is, in Kennedy et al 2017’, [1], words, as follows: "we seem to be very far from a full understanding of where and why moduli occur".

[1] Colley S., Castro A., Kennedy G., Shanbrom C., *A Coarse Stratification of the Monster Tower*, Michigan Mathematical Journal 66 (2017), 855–866.

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An integral formula for G–structures

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Abstract

Equipping an n–dimensional manifold M with a Riemannian metric g is equivalent to the reduction of a frame bundle $L(M)$ to the orthogonal frame bundle $O(M)$, i.e. to the action of a structure group $O(n)$. Assuming moreover that M is oriented we can consider the bundle $SO(M)$ of oriented orthonormal frames. Existence of additional geometric structure can be considered as a reduction of a structure group $SO(n)$ to a certain subgroup G. For example, almost Hermitian structure gives $U(\frac{n}{2})$–structure, almost contact metric structure is just a $U(\frac{n-1}{2}) \times 1$–structure, etc.

If ∇ is the Levi–Civita connection of (M, g) we may measure the defect of ∇ to be a G–connection. This leads to the notion of an intrinsic torsion. If this $(1, 2)$–tensor vanishes (in such case we say that a G–structure is integrable) then ∇ is a G–connection, which implies that the holonomy group is contained in G. We may classify non–integrable geometries by finding the decomposition of the space of all possible intrinsic torsions into irreducible G–modules. This approach was initiated by Gray and Hervella for $U(\frac{n}{2})$–structures [3] and later considered for other structures by many authors. Each, so–called, Gray–Hervella class, gives some restrictions on the curvature.

One possible approach to curvature restrictions on compact G–structures can be achieved by obtaining integral formulas relating considered objects. This has been firstly done, in a general case, by Bor and Hernández Lamoneda [1]. They uses Bochner–type formula for forms being stabilizers of each considered subgroup in $SO(n)$. They obtained integral formulas for $G = U(\frac{n}{2}), SU(\frac{n}{2}), G_2$ and Spin_7 and continued this approach for $Sp(n)Sp(1)$ in [2]. The case $G = U(\frac{n-1}{2}) \times 1$ has been studied later in [4] by other authors.

We show how mentioned formulas can be obtained in a different way. The motivation comes from the work of Walczak [5], where the author considered almost product structures, i.e., complementary orthogonal distributions. The nice feature of our approach is that the main integral formula

$$\frac{1}{2}\int_M s_{\mathfrak{g}^\perp} - s^{\mathrm{alt}}_{\mathfrak{g}^\perp}\,\mathrm{vol}_M = \int_M |\chi|^2 + |\xi^{\mathrm{alt}}|^2 - |\xi^{\mathrm{sym}}|^2\,\mathrm{vol}_M.$$

is valid for any G–structure on closed M for compact $G \subset SO(n)$. Let us roughly describe the approach and all used objects in this formula. We consider, so–called, characteristic vector field $\chi = \sum_i \xi_{e_i} e_i$ induced by the intrinsic torsion ξ and calculate its divergence. $\xi^{\rm alt}$ and $\xi^{\rm sym}$ denote the skew–symmetric and symmetric components of ξ, $\xi^{\rm alt}_X Y = \frac{1}{2}(\xi_X Y - \xi_Y X)$, $\xi^{\rm sym}_X Y = \frac{1}{2}(\xi_X Y + \xi_Y X)$, whereas, $s_{\mathfrak{g}^\perp}$ and $s^{\rm alt}_{\mathfrak{g}^\perp}$ are, in a sense, $\mathfrak{g}^\perp$ components of a scalar curvature. Here, $\mathfrak{g}^\perp$ is the orthogonal complement of the Lie algebra $\mathfrak{g}$ of a Lie group G inside the Lie algebra $\mathfrak{so}(n)$ of the Lie group $SO(n)$. For some Gray–Hervella classes the characteristic vector field vanishes, and then we get point–wise formula relating an intrinsic torsion with a curvature.

We derive explicit integral formulas in the following cases:

- almost Hermitian structures,
- special almost Hermitian structures,
- almost contact metric structures,
- G_2 structures,
- Spin(7) structures.

In the way described above we recover many well known relations. Let us state some of the consequences of the main integral formula (without definitions of used objects):

1. Assume (M, g, J) is closed Hermitian manifold of Gray–Hervella type $\mathcal{W}_4$ such that $s = s^*$, where s is a scalar curvature and s^* is a $*$–scalar curvature. Then M is Kähler (compare [1]).
2. On a closed $SU(n)$–structure of type $\mathcal{W}_1 \oplus \mathcal{W}_5$ we have $\int_M s = 5\int_M s^*$.
3. Assume M is a G_2–structure of type $\mathfrak{X}_1 \oplus \mathfrak{X}_2 \oplus \mathfrak{X}_3$ induced by an endomorphism T. Then, the scalar curvature is given by the formula
$$\frac{1}{6}s = -\frac{3}{2}i_0(T) + 6\sigma_2(T),$$
where $i_0(T)$ and $\sigma_2(T)$ are quadratic invariants of T.
4. The scalar curvature of an Spin(7)–structure induced by the Lee form θ and Λ^3_{48} component of $\delta\Phi$ equals
$$s = \frac{21}{8}|\theta|^2 - \frac{1}{2}|(\delta\Phi)_{48}|^2 + \frac{7}{2}\operatorname{div}\theta.$$

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Pairs of foliations and a conformal invariant

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joint work with Jerzy Kalina and Wojciech Kozłowski

Abstract

Pairs of mutually orthogonal complementary foliations of a Riemanian manifold (M, g) appear naturally in geometry. They appear also in physics. For M being a domain in $\mathbb{R}^3$ let $u : M \to \mathbb{R}$ be the potential of an electric or magnetic field in M. Then the family of equipotential surfaces of u (the first foliation) is orthogonal to the family of lines of the force field (the other foliation). The situation generalizes to pairs of orthogonal foliations defined as the level sets of of a pair of so called conjugate submersions on an arbitrary Riemannian manifold M. The properties of such pairs have been investigated in the context of their conformal capacity in [1] and [2]. Let us also note that pairs of orthogonal distributions, and so, in particular, pairs of foliations, were also investigated by variational methods in the class of Riemannian connections on (M, g) in [4]. Here the geometry of a pair of mutually orthogonal complementary foliations will be examined for a suitable torsion–free connection arising from the Bott connection. The connection has a number of nice geometric properties, however, it is not metric. The metrization leads to a connection with torsion. This metrized connection apprears to be the unique affine connection adapted to each of the foliations such that the endomorphisms of the tangent bundles induced by the partial torsions are both self-conjugate. Investigation of the geometry of the connection leads to a tensor which may be treated as a measure of the "lack of the symmetry" of the Weingarten operator, so it encodes the extrinsic geometry of both foliations. This tensor is also a conformal invariant [3].

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Higher order algebroids and representations up to homotopy

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Abstract

The concept of a higher algebroid, as introduced by M. Jóźwikowski and M. Rotkiewicz, naturally generalizes the notions of an algebroid and a higher tangent bundle. The idea is based on a description of (Lie) algebroids as differential relations of a special kind. My goal is to explain the notion of a higher algebroid in a more standard language, i.e. in terms of some bracket operations and vector bundle morphisms. In order two we end up with representation up to homotopy of (Lie) algebroids.

New geometric structures on 3-manifolds through generalized geometry

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Abstract

Generalized geometry has used Dirac structures [3] and Courant algebroids [6] to offer frameworks where classical structures join each other in some overarching new structure. For instance, generalized complex geometry [5, 4] encompass complex and symplectic structures. But there is more to it: symplectic foliations with a holomorphic transverse structure are also described by a generalized complex structure. In fact, although every generalized complex manifold must be almost complex, there are 4-dimensional examples of generalized complex manifolds that admit neither complex nor symplectic structures [2].

Generalized geometry admits various starting setups [1]. For odd-dimensional manifolds, a very convenient one is B_n-generalized geometry, so labelled by the role played by $\mathrm{SO}(2n+1)$, a group of Lie type B_n. By direct analogy with generalized complex geometry, we define B_n-generalized complex structures [7], or simply B_n-structures, which now encompass cosymplectic and normal almost complex structures, odd-dimensional analogues of symplectic and complex structures. But again, there is much more to it.

In this talk I will give a straightforward introduction to generalized complex geometry using, instead of Dirac structures and Courant algebroids, differential forms and an underlying Clifford action. This is also the shortest path to then introduce and grasp B_n-structures. Finally, I will comment on recent joint work with Joan Porti about 3-manifolds, which includes examples of manifolds admitting a B_3-structure but not a cosymplectic or a normal almost contact one.

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Conformal structures with G_2-twistor distribution

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Abstract

Given a split signature conformal structure on a 4-manifold, there is a naturally defined rank 2 distribution on the circle twistor bundle of selfdual null 2-planes. We will discuss the geometry behind this construction and present a classification of (multiply transtitive) homogeneous 4-dimensional conformal structures for which the symmetry algebra of the rank 2 twistor distribution is the exceptional Lie algebra of type G_2. This is joint work with PawełNurowski (CFT PAS) and Dennis The (UiT). The research leading to the results was supported by the GRIEG project SCREAM.

A geometric journey through the Hamilton-Jacobi theory

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Abstract

In this talk I will present an exhaustive compendium of geometric Hamilton-Jacobi formulations in different geometric backgrounds.

This talk is not only intended for presenting a Hamilton-Jacobi theory on different manifolds, but also for the introduction and review of different geometric structures with their corresponding use in physical phenomena. For example, contact manifolds are presented as a suitable framework for the formulation of thermodynamics, whilst cosymplectic manifolds are a keypoint in the development of time-dependent dynamics. We will contemplate some particular geometric structures: Poisson and contact manifolds, implicit mechanics, Nambu dynamics, field theory, locally conformal formalisms and some discrete counterparts of the mentioned structures. For such structures, we will develop a Hamilton-Jacobi formulation and we will provide examples of the application on the different corresponding backgrounds.

Perturbations of Fefferman spaces over CR three-manifolds

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Abstract

In 1976, Charles Fefferman constructed, in a canonical way, a Lorentzian conformal structure on a circle bundle over a given strictly pseudoconvex Cauchy-Riemann (CR) manifolds of hypersurface type. It is also known, notably through the work of Sir Roger Penrose and his associates, and that of the Warsaw group led by Andrzej Trautman, that CR three-manifolds underlie Einstein Lorentzian four-manifolds whose Weyl tensors are said to be algebraically special. I will show how these two perspectives are related to each other, by presenting modifications of Fefferman's original construction, where the conformal structure is "perturbed" by some semi-basic one-form, which encodes additional data on the CR three-manifold. Our setup allows us to reinterpret previous works by Lewandowski, Nurowski, Tafel, Hill, and independently, by Mason. Metrics in such a perturbed Fefferman conformal class whose Ricci tensor satisfies certain degeneracy conditions, are only defined off sections of the Fefferman bundle, which may be viewed as "conformal infinity". The prescriptions on the Ricci tensor can then be reduced to differential constraints on the CR three-manifold in terms of a "complex density" and the CR data of the perturbation one-form. One such constraint turns out to arise from a non-linear, or gauged, analogue of a second-order differential operator on densities. A solution thereof provides a criterion for the existence of a CR function and, under certain conditions, for CR embeddability. This talk is based on arxiv:2303.07328.

Totally geodesic submanifolds and homogeneous spheres

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Abstract

The classification of transitive Lie group actions on spheres was obtained by Borel, Montgomery, and Samelson in the forties. As a consequence of this, it turns out that apart from the round metric there are other Riemannian metrics in spheres that are invariant under the action of a transitive Lie group. These other homogeneous metrics in spheres can be constructed by modifying the metric of the total space of the complex, quaternionic, or octonionic Hopf fibration in the direction of the fibers.

In this talk, I will report on a joint work with Carlos Olmos (Universidad Nacional de Córdoba), where we classified totally geodesic submanifolds in Hopf-Berger spheres. These are those Riemannian homogeneous spheres obtained by rescaling the round metric of the total space of Hopf fibrations by a positive factor in the direction of the fibers.

Hard Lefschetz Property for isometric flows and S^3–actions

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joint work with José Ignacio Royo Prieto
and Martin Saralegi-Aranguren

Abstract

The Hard Lefschetz Property (HLP) is an important property which has been studied in several categories of the symplectic world. For Sasakian manifolds, this duality is satisfied by the basic cohomology (so, it is a transverse property). A new version of the HLP has been recently given in terms of duality of the cohomology of the manifold itself. Both properties were recently proved to be equivalent in the case of K-contact flows. We show that the HLP is naturally defined for the more general category of isometric flows, and for the category of almost-free S^3–actions, which generalizes the rich properties of the 3-Sasakian manifolds. We also show that both versions of the HLP (transversal and global) are equivalent for isometric flows and for certain almost-free S^3–actions.

Variational problems and methods in extrinsic geometry of distributions

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Abstract

Extrinsic geometry of a distribution (subbundle of the tangent bundle) on a manifold can be described by second fundamental forms and integrability tensors of that distribution and its orthogonal complement. Using these geometric objects to define functionals of Riemannian metric, we characterize contact metric structures as critical points of one of them [1], examine component of the scalar curvature determined by two non-complementary orthogonal distributions [1], and obtain formulas describing changes of geometry of a foliation along a vector field projectable along its leaves.

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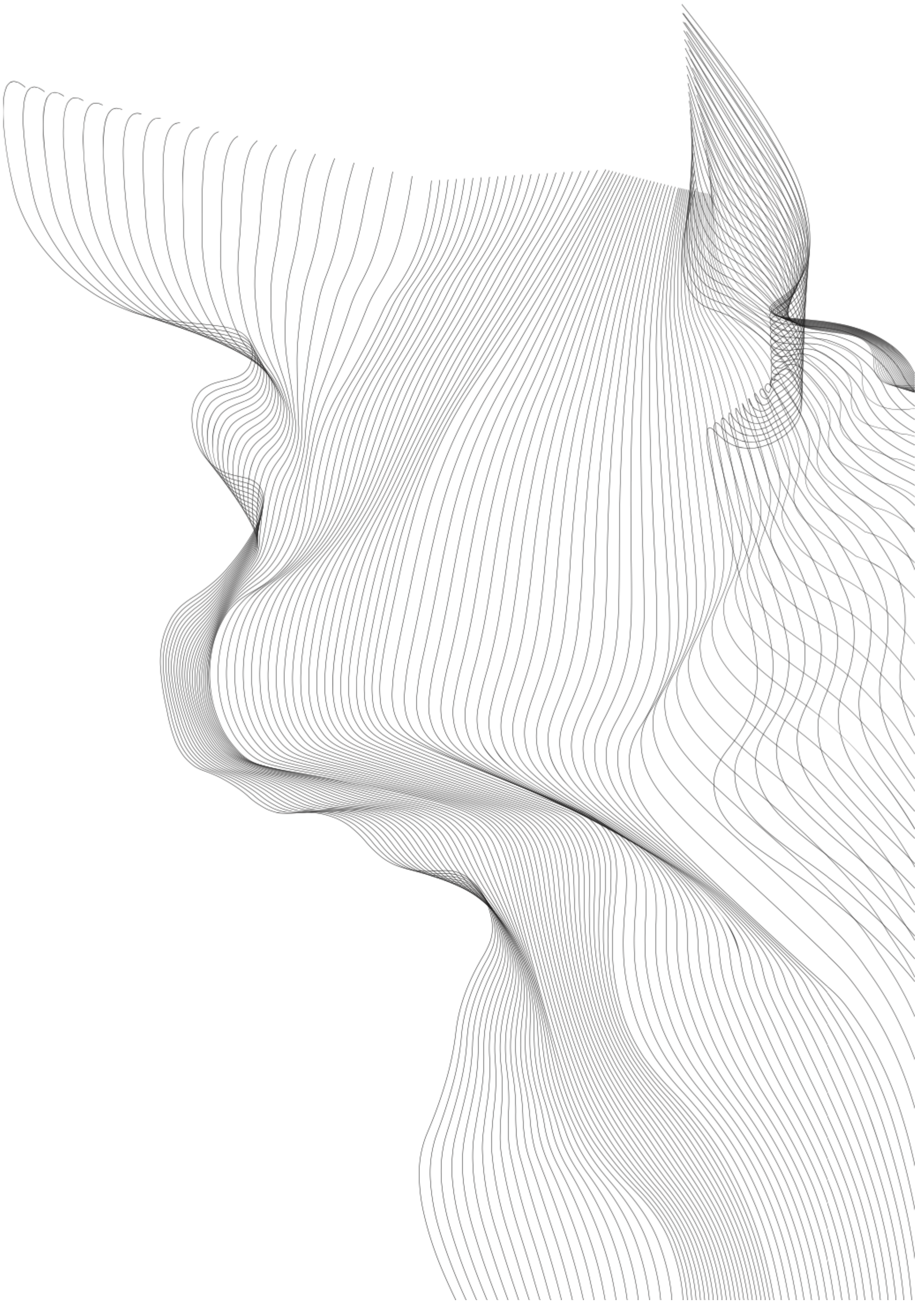

Thematic section

DDS

Discrete Dynamical Systems

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SCHEDULE OF THE SECTION
Discrete Dynamical Systems

- Wednesday – September 6th

  12:00–12:30 Grzegorz Świątek, *Coefficients of the Riemann map for the Mandelbrot Set Complement*

  12:30–13:00 Xavier Jarque, *Connectivity of the basin of attraction of fixed points for some root finding algorithms*

  13:00–13:30 Piotr Oprocha, *On planar attractors and inverse limits*

- Thursday – September 7th

  12:00–12:30 Bartomeu Coll, *Asymptotic dynamic of a difference equation with a parabolic fixed point*

  12:30–13:00 Feliks Przytycki, *Iteration of rational maps on the Riemann sphere: geometric pressures and dimensions*

  14:00–14:30 Andrzej Biś, *Entropy functions for semigroup actions*

  14:30–15:00 Krzysztof Barański, *Prediction of dynamical systems*

  15:00–15:30 Marc Jorba-Cuscó, *Recurrent motion in discrete predator-prey models*

  15:30–16:00 Robert Florido-Llinàs, *Towards an atlas of wandering domains for a family of Newton maps*

  coffee break

  16:30–17:00 Krzysztof Leśniak, *Toward IFSs with non-metrizable attractors*

  17:00–17:30 Anna Jové, *Density of periodic points in boundaries of Fatou components*

  17:30–18:00 Frank Llovera, *Lorenz-like maps in classification of spike-patterns in a map-based neuron model*

- Friday – September 8th

  10:45–11:15 Michał Misiurewicz, *The Real Teapot*

  11:15–11:45 Marina Gonchenko, *Homoclinic tangencies in area-preserving maps*

  11:45–12:15 David Rojas, *Characterization of the tree cycles with minimum positive entropy for any period*

  12:15–12:45 Salvador Borrós-Cullell, *Computing regularities of invariant objects using wavelets*

  14:30–15:00 Miguel Ángel Fernandez Sanjuán, *Partial Control and Beyond: Controlling Chaotic Transients with the Safety Function*

  15:00–15:30 Josep Sardanyés, *Navigating the Unseen: transients and ghosts close to bifurcations*

  15:30–16:00 Janina Kotus, *Lyapunov exponent for non-ergodic meromorphic functions*

Prediction of dynamical systems

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joint work with Yonatan Gutman and Adam Śpiewak
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Abstract

Schroer, Sauer, Ott and Yorke conjectured in 1998 that the Takens delay embedding theorem can be improved in a probabilistic context. More precisely, their conjecture states that if μ is a natural measure for a smooth diffeomorphism of a Riemannian manifold and k is greater than the dimension of μ, then k time-delayed measurements of a one-dimensional observable are generically sufficient for a predictable reconstruction of μ-almost every initial point of the original system. This reduces by half the number of required measurements, compared to the standard (deterministic) setup. We prove the conjecture for all Lipschitz systems (also non-invertible) on compact sets with an arbitrary Borel probability measure and establish an upper bound for the decay rate of the measure of the set of points where the prediction is subpar. We also prove general time-delay prediction theorems for locally Lipschitz or Hölder systems on Borel sets in Euclidean space.

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Entropy functions for semigroup actions

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based on joint work with Maria Carvalho,
Miguel Mendes and Paulo Varandas

Abstract

Using methods from Convex Analysis, for each generalized pressure function we define an upper semi-continuous affine entropy-like map, establish an abstract variational principle for both countably and finitely additive measures and prove that equilibrium states always exist. We study the thermodynamic formalism of continuous actions of semigroups generated by continuous self-maps or homeomorphisms of a compact metric space X. This setting comprises finitely generated semigroups, countable sofic groups and uncountable groups endowed with a reference probability measure. For each topological pressure operator associated to these actions we provide both an affine, upper semi-continuous entropy-like map, whose domain is the set of Borel probability measures on X, and a variational principle whose maximum is always attained.

Computing regularities of invariant objects using wavelets

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Abstract

This is a continuation of [3]. We want to study the family of skew products over $\mathbb{S}^1 \times \mathbb{R}$

$$\begin{pmatrix} \theta_{n+1} \\ x_{n+1} \end{pmatrix} = \begin{pmatrix} \theta_n + \omega \mod 1 \\ F_{\sigma,\varepsilon}(\theta_n, x_n) \end{pmatrix},$$

where $\omega \in \mathbb{R} \backslash \mathbb{Q}$ and $F_{\varepsilon,\sigma} : \mathbb{R}^2 \to \mathbb{R}$ depends on two parameters ε and σ. Depending on the choice of F the system may have a Strange Non-Chaotic Attractor (SNA). We have devised a method to measure the *strangeness* of the invariant object via its *wavelet expansion*.

$$\varphi \sim a_0 + \sum_{j=0}^{\infty} \sum_{n=0}^{2^j-1} \langle \varphi, \psi_{-j,n}^{\text{PER}} \rangle \psi_{-j,n}^{\text{PER}}. \tag{BC}$$

where $\psi_{-j,n}^{\text{PER}}$ denotes a periodized Daubechies wavelet with p vanishing moments. However, this is not an easy task, mainly due to the fact that Daubechies wavelets do not have a closed expression.

Once an expression such as in (BC) is archived, the regularity of the attractor can be approximated through Besov Spaces $\mathcal{B}_{\infty,\infty}^s$[4], since from the wavelet coefficients $\langle \varphi, \psi_{-j,n}^{\text{PER}} \rangle$ one can determine the value of s such that $\varphi \in \mathcal{B}_{\infty,\infty}^s$.

Mainly we will present the results obtained by testing them on the system studied by Keller [1] and give some new results concerning the Nishikawa-Kaneko system [2].

[1] Keller G., *A note on strange nonchaotic attractors*, Fundamenta Mathematicae 151 (1996), no. 2, 139–148.

[2] Nishikawa T., Kaneko K., *Fractalization of a torus as a strange non-chaotic attractor*, Physical Review E (1996).

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Asymptotic dynamic of a difference equation with a parabolic fixed point

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Abstract

The aim of this work is the study of the asymptotic dynamical behaviour, of solutions that approach parabolic fixed points in difference equations. In one dimensional difference equations, we present the asymptotic development for positive solutions tending to the fixed point. For higher dimensions, through the study of two families of difference equations in the two and three dimensional case, we take a look at the asymptotic dynamic behaviour. To show the existence of solutions we rely on the parametrization method.

[1] Coll B., Gasull A., Prohens R., *Asymptotic Dynamics of a Difference Equation with a Parabolic Equilibrium*, Qualitative Theory of Dynamical Systems 19 (2020), no. 70.

Towards an atlas of wandering domains for a family of Newton maps

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joint work with N. Fagella

Abstract

We study the class of transcendental meromorphic functions f which are semiconjugate via the exponential to finite-type maps g in Bolsch's class [1]. Here we investigate the coexistence of wandering domains and attracting invariant basins for one-parameter families of Newton's methods f, in close relation to the forward orbit of free critical points of g, and the logarithmic lifting method for periodic Fatou components introduced by Herman [2].

[1] Bolsch A., *Iteration of meromorphic functions with countably many essential singularities*, doctoral dissertation, Technischen Universität Berlin, 1997.

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Homoclinic tangencies in area-preserving maps

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Abstract

We study bifurcations in area-preserving maps with homoclinic tangencies. We consider C^r-smooth maps ($r \geqslant 3$) with a saddle fixed point whose stable and unstable invariant manifolds have a quadratic or cubic tangency at the points of some homoclinic orbit and we study bifurcations of periodic orbits near the homoclinic tangencies in perturbed area-preserving maps. Every point of these orbits can be considered as a fixed point of the so-called first return maps defined along the tangency. In the case of a quadratic homoclinic tangency, we prove the existence of cascades of generic elliptic periodic orbits for one and two parameter unfoldings. In the case of a cubic homoclinic tangency, we establish the structure of bifurcation diagram in two parameter unfoldings.

Connectivity of the basin of attraction of fixed points for some root finding algorithms

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Abstract

In this talk we discuss the connectivity of the basin of attraction of fixed points for some root finding algorithms. It is known that the Julia set of a Newton map (applied to either polynomial or entire map) is connected, and so, all Fatou components are simply connected domains (in particular the immediate basins of attraction of the fixed points associated to the zeroes of the polynomial (or entire map)). Moreover those basins are unbounded. A key argument in the proof is the non existence of weakly repelling finite fixed points. However this property is not satisfied for other root finding algorithms like Halley or Traub. We will discuss some connectivity as well as unboundedness results for those methods.

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[2] Paraschiv D. A. , *Newton-like components in the Chebyshev-Halley family of degree n polynomials*, Mediterranean Journal of Mathematics 20 (2023), no. 149.

Recurrent motion in discrete predator-prey models

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joint work with Lluís Alsedà and Tomás Lázaro

Abstract

Predator-prey models are mathematical tools used in ecology to study how predators and prey influence each other's population sizes over time. These models show that when prey populations are high, predator populations tend to increase too, leading to a decrease in prey. As prey declines, predator populations also decrease, starting the cycle again. These models help us understand the dynamics of ecosystems but may not capture all complexities in real-life interactions.

In this work we consider the following discrete dynamical system:

$$\begin{cases} \bar{x} = \mu x(1 - x - y), \\ \bar{y} = \beta xy. \end{cases}$$

Here, the preys x grows logistically with an intrinsic reproduction rate μ and the predators y increase their population numbers at rate β. Although very simple, this model displays very rich dynamics (see [1]). We discuss recurrent motion (periodic, quasi-periodic and chaotic orbits) and identify trajectories which are not stable but behave as they were for some amount of time.

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Density of periodic points in boundaries of Fatou components

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joint work with N. Fagella

Abstract

This talk concerns holomorphic dynamics, and the study of Fatou and Julia sets. In particular, we will address the problem of finding periodic points in the boundary of attracting and parabolic basins, and some types of Baker domains. For rational maps, F. Przytycki and A. Zdunik proved that periodic points are always dense in the boundary of attracting or parabolic basins. New ideas and techniques to work with transcendental functions will be provided.

Lyapunov exponent for non-ergodic meromorphic functions

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Abstract

Levin, Przytycki and Shen proved for a polynomial map $f_c(z) = z^d + c$, $d \geqslant 2$ and $c \in \mathbb{C}$ with Julia set $J(f_c)$ of positive measure that for a.e. $z \in J(f_c)$ the Lyapunov exponent $\chi_s(z) = 0$. Dobbs proved that transcendental entire this is result is not true. We will show that it does not occur also for transcendental meromorphic functions with poles.

Toward IFSs with non-metrizable attractors

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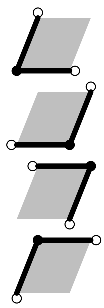
joint work with Magdalena Nowak

Abstract

We are going to construct some iterated function systems (IFS) with non-metrizable attractors, like a split square; see the picture on the right.



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Lorenz-like maps in classification of spike-patterns in a map-based neuron model

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joint work with Piotr Bartłomiejczyk
and Justyna Signerska-Rynkowska

Abstract

We study the well-known map-based model of neuronal dynamics introduced in 2007 by Courbage, Nekorkin and Vdovin, important due to various medical applications. We also review and extend some of the existing results concerning β-transformations and (expanding) Lorenz mappings. Then we apply them for deducing important properties of the spike-trains generated by the CNV model and explain their implications for the neuron behaviour. In particular, using recent theorems of rotation theory for Lorenz-like maps, we provide a classification of periodic spiking patterns in this model.

[1] Bartlomiejczyk P., Llovera F., Signerska-Rynkowska J., *Spike patterns and chaos in a map-based neuron model*, International Journalof Applied Mathematics and Computer Science 33 (2023), no. 3.

The Real Teapot

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joint work with Lluis Alseda, Jozef Bobok, Lubomir Snoha

Abstract

In his last paper, William Thurston defined the Master Teapot as the closure of the set of pairs (z, s), where s is the slope of a tentmap T_s with the turning point periodic, and a complex number z isa Galois conjugate of s. In this case $1/z$ is a zero of the kneading determinant of T_s. We remove the restriction that the turning point is periodic, and sometimes look beyond tent maps.

However, we restrict our attention to zeros $x = 1/z$ in the real interval $(0, 1)$. By the results of Milnor and Thurston, the kneading determinant has such a zero if and only if the map has positive topological entropy. We show that the first (smallest) zero is simple, but among other zeros there may be multiple ones. We describe a class of unimodal maps, so-called R-even ones, whose kneading determinant has only one zero in $(0, 1)$. In contrast with this, we show that generic mixing tent maps have kneading determinants with infinitely many zeros in $(0, 1)$. We prove that the second zero in $(0, 1)$ of the kneading determinant of a unimodal map, provided it exists, is always larger than or equal to $\sqrt[3]{1/2}$ and if the kneading sequence begins with $RL^N R$, $N \geqslant 2$, then the best lower bound for the second zero is in fact $\sqrt[N+1]{1/2}$. We also investigate (partially numerically) the shape of the *Real Teapot*, consisting of the pairs (s, x), where x in $(0, 1)$ is a zero of the kneading determinant of T_s, and $s \in (1, 2]$.

On planar attractors and inverse limits

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joint work with Jernej Činč

Abstract

A very useful technique called BBM (Brown-Barge-Martin), incorporates inverse limits and natural extensions of the underlying bonding maps to embed attractors in manifolds. The original idea goes back to the paper of Barge and Martin, where the authors constructed strange attractors from a wide class of inverse limits. One of the crucial steps for this technique to work is the usage of Brown's approximation theorem. Recently, this technique was extended to produce a parameterized family of strange attractors. In this talk we will present a few possible applications of BBM technique in construction of concrete examples.

Iteration of rational maps on the Riemann sphere: geometric pressures and dimensions

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Abstract

I will pose some questions on

1. Geometric pressure via periodic orbits.
2. Dimension of Julia sets as the first zero of the geometric pressure.
3. Geometric coding trees,

Characterization of the tree cycles with minimum positive entropy for any period

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joint work with Francesc Mañosas and David Juher

Abstract

The notion of *pattern* plays a central role in the theory of topological and combinatorial dynamics. Consider a family $\mathcal{X}$ of topological spaces (for instance the family of either all closed intervals of the real line or all trees or all graphs or compact surfaces, etc) and the family $\mathcal{F}_\mathcal{X}$ of all maps $\{f : X \to X : X \in \mathcal{X}\}$ satisfying a given restriction (continuous maps, homeomorphisms, etc). Given a map $f : X \to X$ in $\mathcal{F}_\mathcal{X}$ which is known to have a finite invariant set P, the *pattern of P in $\mathcal{F}_\mathcal{X}$* is the equivalence class $\mathcal{P}$ of all maps $g : Y \to Y$ in $\mathcal{F}_\mathcal{X}$ having an invariant set $Q \subset Y$ that, at a combinatorial level, behaves like P. That is, the relative positions of the points of Q inside Y are the same as the relative positions of P inside X, and the way these positions are permuted under the action of g coincides with the way f acts on the points of P. In this case, it is said that every map g in the class *exhibits* the pattern $\mathcal{P}$. If in particular P is a periodic orbit of f, the pattern $\mathcal{P}$ is said to be *periodic*.

In this talk we deal with patterns of invariant sets of continuous maps defined on trees (simply connected graphs). We consider, for any integer $n \geqslant 3$, the set Pos_n of all n-periodic tree patterns with positive topological entropy. We explicitly construct an n-periodic tree pattern $\mathcal{Q}_n$ whose entropy is minimum in Pos_n.

Partial Control and Beyond: Controlling Chaotic Transients with the Safety Function

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joint work with G. Alfaro and R. Capeáns from URJC, Spain

Abstract

A new control algorithm based on the partial control method has been developed. The general situation we are considering is an orbit starting in a certain phase space region Q having a chaotic transient behavior affected by a bounded noise, so that the orbit will definitely escape from Q in an unpredictable number of iterations. Thus, the goal of the algorithm is to control in a predictable manner when to escape. While partial control has been used as a way to avoid escapes, here we want to adapt it to force the escape in a controlled manner. We have introduced new tools such as escape functions and escape sets that once computed makes the control of the orbit straightforward. The partial control method aims to avoid the escape of orbits from a phase space region Q where the transient chaotic dynamics takes place. The technique is based on finding a special subset of Q called the safe set. The chaotic orbit can be sustained in the safe set with a minimum amount of control. We have developed a control strategy to gradually lead any chaotic orbit in Q to the safe set by using the safety function. With the technique proposed here, the safe set can be converted into a global attractor of Q. In addition, we deal with the Hénon and the Lozi maps for a choice of parameters where they show transient chaos, and we compute their safety functions showing the strong dependence of the safety function with the strength of the bounded noise affecting the maps, drastically impacting the controlled orbits.

[1] Alfaro G., Capeáns R., Sanjuán M.A.F., *Forcing the escape: Partial control of escaping orbits from a transient chaotic region*, Nonlinear Dynamics 104 (2021), 1603–1612.

[2] Capeáns R., Sanjuán M.A.F., *Beyond partial control: Controlling chaotic transients with the safety function*, Nonlinear Dynamics 107 (2022), 2903–2910.

[3] Capeáns R., Sanjuán M.A.F., *Controlling chaotic transients in the Hénon and the Lozi map with the safety function*, Journal of Difference Equations and Applications (2023).

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Navigating the Unseen: transients and ghosts close to bifurcations

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Abstract

It is known that transients towards attractors become longer close to bifurcations. This phenomenon, which is found in local and global bifurcations, can have deep implications in multitude of dynamical systems such as ecosystems. Such implications range from delays in population collapses or in difficulties in the recovery of ecosystems with multiple stable states. In this talk we will focus on so-called ghost transients which typically appear after saddle-node bifurcations. We will focus on the properties of these transients investigated in discrete-time and discrete-time-space ecological models. Other examples of ghost manifolds will be introduced.

[1] Canela J., Fagella N., Alsedà Ll., Sardanyés J., *Dynamical mechanism behind ghosts unveiled in a map complexification*, Chaos, Solitons & Fractals 156 (2022).

[2] Dai L., Vorselen D., Korolev K.S., Gore J., *Generic Indicators for Loss of Resilience Before a Tipping Point Leading to Population Collapse*, Science 336 (2012), no. 6085, 1175–1177.

[3] Duarte J., Januário C., Martins N., Sardanyés J., *On chaos, transient chaos and ghosts in single population models with Allee effects.* Nonlinear Analysis Series B 13 (2012), no. 4, 1647–1661.

[4] Sardanyés J., Solé R., *Bifurcations and phase transitions in spatially extended two-member hypercycles*, Journal of Theoretical Biology 234 (2006), no. 4, 468–482.

[5] Trickey S.T., Virgin L.N., *Bottlenecking phenomenon near a saddle-node remnant in a Duffing oscillator*, Physics Letters A 248 (1998), no. 2, 185–190.

Coefficients of the Riemann map for the Mandelbrot Set Complement

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joint with Grzegorz Siudem of *Warsaw Tech.*

Abstract

We propose a numerical algorithm for the approximate computation of the coefficients of the standardized Riemann map for the complement of the Mandelbrot set. Let $f_c(z) = z^2 + c$ be the standard representation of the quadratic family and $\mathcal{M}$ the Mandelbrot set and $\Psi : \overline{\mathbb{C}}\backslash\overline{\mathbb{D}(0,1)} \to \overline{\mathbb{C}}\backslash\mathcal{M}$ is normalized Riemann map. We start with the Fourier transform of the normalized Riemann map given with the formula $\Psi^{-1}(c) = \lim_{n\to\infty} \sqrt[2^n]{f_c^n(c)}$. The application of Fourier transform reduces the problem to the simple relation, solvable with Newton's method. Replacing the full Fourier series equivalent to the power series expansion by a discrete Fourier transform introduces a certain error, but it is easy to estimate and control. The method has enabled us to compute 2^n coefficients of the Riemann map in $O\left(n2^n\right)$ operations and obtain coefficients up to $n = 26$ on standard hardware and high accuracy. Collected data suggest that the coefficients decay fairly quickly but insufficiently to make the sequence summable and certain other conjectures were validated or posed. The algorithm is easily parallelizable in the most computationally demanding part using the Newton's method and, less easily, using a fast parallel implementation of the Fourier transform.

[1] Bittner D., Cheong L. et al., *New aproximations for the area of the Mandelbrot set*, Involve 10 (2017).

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Thematic section:

DDE

Dynamics of Differential Equations

ORGANIZERS:

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SCHEDULE OF THE SECTION
Dynamics of Differential Equations

- Monday – September 4th

  16:00–16:30 Andrew Clarke, *Why are inner planets not inclined?*

  16:30–17:00 Óscar Rodríguez, *Dynamical study of Hilda asteroids through quasi-periodic solutions*

  coffee break

  17:30–18:00 Aleksander Pasiut, *Oscillatory orbits to collision in the planar circular restricted three body problem*

  18:00–18:30 Piotr Zgliczyński, *Shadowing of non-transversal heteroclinic chains*

  18:30–19:00 Amadeu Delshams, *Polynomial normal forms for ODEs preserving some dynamical structures*

- Tuesday – September 5th

  14:30–15:00 José Lamas Rodríguez, *Parabolic ejection & collision orbits for the restricted planar circular three body problem*

  15:00–15:30 Rodrigo G. Schaefer, *Arnold Diffusion via Scattering maps: A geometrical mechanism to detect global instability*

  15:30–16:00 Maciej Capiński, *Characterising blenders via covering relations and cone conditions*

  coffee break

  16:30–17:00 Robert Szczelina, *A geometric method for computer assisted proofs in delay differential equations*

  17:00–17:30 Pau Martín, *Chaotic scattering of He atoms off a Cu surface with corrugated Morse potential*

  17:30–18:00 Małgorzata Moczurad, *Central configurations for $N + k$ body problem*

- Thursday – September 7th

  14:00–14:30 Chen Xuan, *The maths of a photo induced hydrogel swimming robot: nonsmooth forcing dynamics*

  14:30–15:00 Piotr Kalita, *On unbounded attractors for dynamical systems*

  15:00–15:30 Juan Garcia Fuentes, *Geometrical description of forwards attractors for non-autonomous Lotka-Volterra systems*

  15:30–16:00 Maria Przybylska, *TBA*

  coffee break

  16:30–17:00 Andrzej Maciejewski, *TBA*

Why are inner planets not inclined?

Andrew Clarke

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and Jacques Fejoz, *Université Paris Dauphine, Observatoire de Paris*
and Marcel Guardia, *Universitat de Barcelona, Centre de Recerca Matemàtica*

Abstract

Consider the Newtonian 4-body problem, in the regime where 3 bodies (the planets) revolve on near-elliptical orbits around the other body (the sun). A long-held belief, culminating in the XVIII century in the first stability theorem of Laplace and Lagrange, is that the semimajor axes are stable. Assuming the initial conditions of the semimajor axes are of different orders, and that there is a large mutual inclination between planets 1 and 2, we prove that there are orbits of the 4-body problem where the semimajor axis of planet 3 can follow any itinerary, with arbitrary precision. In addition, along such orbits, we can make the normalised angular momentum vector of planet 2 follow any itinerary, as well as the eccentricity of the orbital ellipse of planet 2, again with arbitrary precision. For example, planet 2 may flip from prograde to retrograde nearly- horizontal revolutions. These orbits constitute a counterexample to the first stability theorem of Laplace and Lagrange. Moreover, as a consequence of the proof, the non-recurrent set of any finite-order secular normal form accumulates on circular motions, thus proving a weak form of a celebrated conjecture of Herman.

Polynomial normal forms for ODEs preserving some dynamical structures

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joint work with Piotr Zgliczyński

Abstract

We prove the polynomial normal form theorem for flows that preserve some dynamical structures. These new structures are the preservation of some invariant subspaces and the block diagonal structure of the variational equations along them, which are important for the shadowing of nontransverse heteroclinic chains.

Geometrical description of forwards attractors for non-autonomous Lotka-Volterra systems

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joint work with José Antonio Langa, Piotr Kalita and Antonio Suárez,
fully developed in [3]

Abstract

The autonomous Lotka-Volterra model is used to study the evolution of species population from an ecosystem, and the full characterization of the asymptotic behavior of their solutions is well known. In particular, it is possible to give conditions on the coefficients which guarantee the existence of a globally asymptotically stable equilibrium point, in which all the species are present (known as a permanence solution), or, on the contrary, conditions to obtain a stable equilibrium point which possess one or more species extinct. Futhermore, one can construct the full structure of the global attractor associated through the heteroclinics conections between equilibrium points [1]. Passing these results to the non-autonomous situation, i.e. when the parameters are depending on time, is non-trivial, and in general, it is rare to get time dependent invariant compact attracting sets when time goes to $+\infty$. Based on the works of Lazer and Ahmad [1], [2], and Redheffer [4], we consider a non-autonomous Lotka-Volterra system where the species cooperate between them. We give sufficient conditions to obtain the existence of a globally asymptotically stable solutions, in both the permanence and extinction situations. Once obtained the stable solution, we obtain the exact geometrical structure of the forwards non-autonomous attractor by constructing the heteroclinic connections between the globally stable solution and the semistables ones.

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[2] Ahmad S., Lazer A., *Necessary and sufficient average growth in a Lotka–Volterra system*, Nonlinear Analysis 34 (1998), 191–228.

[3] Garcia-Fuentes J., Langa J.A., Kalita P., Suárez A., *Characterization of attractors for non-autonomous Lotka-Volterra cooperative systems*, arXiv:2301.04955 (2023).

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[5] Takeuchi Y., *Global asymptotic dynamical properties of Lotka-Volterra systems*, World Scientific Publishing, 1996.

On unbounded attractors for dynamical systems

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Abstract

If the semigroup is slowly non-dissipative, i.e., its trajectories can diverge to infinity as time tends to infinity, one still can study its dynamics via the approach by the unbounded attractors - the counterpart of the classical notion of global attractors. We continue the development of this theory started by Chepyzhov and Goritskii in [4] and extended recently in [1, 2, 3]. We provide the abstract results on the unbouded attractor existence in slowly non-dissipative setting for autonomous and non-autonomous situation. The abstract theory that we develop is illustrated by the analysis of the problem governed by the equation

$$u_t = Au + f(u),$$

as well as its non-autonomous counterpart. In particular, using the notion of inertial manifold and graph transform approach and the Lyapunov-Perron approach, we provide the criteria under which the unbounded attractor coincides with the graph of the Lipschitz function, or becomes close to the graph of the Lipschitz function for large argument. At the same time, we derive the new non-autonomous spectral gap-type conditions which guarantee the existence of non-autonomous inertial manifolds in the unbounded attractor setting.

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[2] Bortolan M.C., Fernandes J., *Sufficient conditions for the existence and uniqueness of maximal attractors for autonomous and nonautonomous dynamical systems*, Journal of Dynamics and Differential Equations (2022), doi: 10.1007/s10884-022-10220-8.

[3] Bortolan M.C., Fernandes J., Kalita P., *On unbounded attractors in dynamical systems*, in preparation.

[4] Chepyzhov V.V., Goritskii A.Yu., *Unbounded attractors of evolution equations*, Advances in Soviet Mathematics 10, 85—128, American Mathematical Society, 1992.

Parabolic ejection & collision orbits for the restricted planar circular three body problem

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Abstract

We consider the restricted planar circular three body problem (RPC3BP), which describes the motion of a massless body under the attraction of other two bodies, the primaries, which describe circular orbits around their common center of mass located at the origin.

In a suitable system of coordinates, this system is Hamiltonian with two degrees of freedom, whose conserved energy is usually called the Jacobi constant. In such system, we are interested in solutions of the RPC3BP called *ejection-collision orbits*, i.e., solutions that depart from the big primary at some time t_0 and collide with it at some instant t_1.

In this talk I will explain how to construct arbitrarily large ejection-collision orbits for small values of the mass ratio. To this end, we show that, for small values of the mass ratio and the Jacobi constant, there exist transverse intersections between the stable (unstable) manifold of *infinity* and the unstable (stable) manifold of collision.

Close to such transverse intersections, we prove the existence of a sequence of ejection-collision orbits that travel arbitrarily far away. Moreover, using a similar argument, we prove the existence of a sequence of forward and backward periodic parabolic orbits that travel close to collision too. Finally, we also prove the existence of periodic orbits that travel close to collision and arbitrarily far away.

Chaotic scattering of He atoms off a Cu surface with corrugated Morse potential

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Abstract

We consider the motion of an helium atom bouncing off a copper surface. We assume that the He atom motion takes place on a plane. We denote by $(x, z) \in \mathbb{R}^2$ the position of the He atom, where x and z are the horizontal and vertical displacements, respectively. Let (p_x, p_z) be the conjugate momenta. We assume that the interaction of the He atom with the copper surface is modeled by a corrugated Morse potential, where the corrugation represents the presence of the copper atoms in the surface. More concretely, we consider the Hamiltonian

$$H_{\mathrm{CM}}(x, z, p_x, p_z) = \frac{1}{2m}(p_x^2 + p_z^2) + V_{\mathrm{M}}(z) + V_{\mathrm{C}}\left(\frac{2\pi x}{a}, z\right), \qquad \text{(M1)}$$

where

$$\begin{aligned} V_{\mathrm{M}}(z) &= De^{-\alpha z}(e^{-\alpha z} - 2), \\ V_{\mathrm{C}}(\theta, z) &= De^{-2\alpha z} V(\theta), \\ V(\theta) &= \sum_{n \geqslant 1} r_n \cos(n\theta) + s_n \sin(n\theta). \end{aligned}$$

The coefficients r_n and s_n are determined experimentally. Their values are

$$r_1 = 0.06, \qquad r_2 = 0.008, \qquad r_n = 0, \quad n \geqslant 3, \qquad s_n = 0, \quad n \geqslant 1,$$

$D = 6.35$ meV, $a = 3.6$ Å, $\alpha = 1.05$ Å^{-1}. In particular, $r_1 > 0$. In the present paper, our only requirement on V is that it is an analytic even function.

The purpose of this paper is twofold. On the one hand, we prove presence of chaos in some parts of the phase space of (M1). Here, the

notion of chaos is the one introduced by Smale and it is based on the presence of a *Smale horseshoe* and a conjugation with the shift on a space of symbols. It is worth to remark that the set where chaos takes place is a hyperbolic set.

On the other, we prove the existence of *oscillatory orbits*, that is, solutions $(x(t), z(t))$ of (M1) with the property that $\limsup_t z(t) = \infty$ and $\liminf_t z(t) < \infty$, that is, solutions such that go higher and higher but always go back again to a finite distance of the Cu surface.

Central configurations for $N + k$ body problem

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joint work with Piotr Zgliczyński

Abstract

We study the problem of planar central configurations with N heavy bodies and k bodies with arbitrary small masses.

We derive the equation describing the limit of light masses going to zero, which can be seen as the equation for central configurations in the anisotropic plane. Using computer rigorous computations we compute

all central configurations for $N = 2$ and $k = 3, 4$ and for the derived limit problem. We show that the results are consistent.

Oscillatory orbits to collision in the planar circular restricted three body problem

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Abstract

In our work, we present the computer assisted proof of the existence of infinite number of periodic orbits of arbitrarily long period that approach one of the primary masses of the system arbitrarily close. In order to achieve our goal we make use of topological tools (such as covering relations) and we perform rigorous interval computations with CAPD package. The proof is based on performing the Levi-Civita regularization, showing the existence of the particular orbits with numerical and topological tools and proving that the orbits remain valid when switching back to the non-regularized system.

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Dynamical study of Hilda asteroids through quasi-periodic solutions

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and Begoña Nicolás, *Universidade de Santiago de Compostela, Dept. Matemáticas*

Abstract

The Hilda family of asteroids is a group of more than 5000 asteroids located beyond the main asteroid belt of our Solar System, but within Jupiter's orbit. They are known to have mean motion in a $3:2$ orbital resonance with Jupiter and to describe orbits that seem to successively approach three Lagrangian Points, L_3, L_4 and L_5, of the Sun-Jupiter system.

Our aim in this work is to analyse Hilda's behaviour from a dynamical systems ap- proach, by studying their orbits within Sun-Jupiter Circular Restricted Three Body Prob- lem (CRTBP) and Elliptical Restricted Three Body Problem (ERTBP), both in the planar case. The reason for studying both models is to analyse the level of importance of Jupiter eccentricity in this particular application.

Our analysis starts by selecting those asteroids in the JPL database with orbital elements of the Hilda category, although focusing on those with low inclination. The database provides the coordinates of the asteroids in an inertial ecliptical reference frame, with the origin set at the solar system center of mass. Then, a change of coordinates for the ephemeris of these asteroids is needed in order to have them in the CRTBP or ERTBP Sun-Jupiter systems. This (non trivial) change of coordinates is defined through the instantaneous orbital elements of the Sun and Jupiter and it will be detailed in the presentation.

Once we have the coordinates of the Hilda asteroids in our mathematical models, we are in position of computing numerically the periodic and quasi-periodic orbits that are assumed to be responsible of their motion. In order to compute these quasi-periodic orbits (also known as invariant

tori), we make use of temporal or spatial Poincarè sections. Some comparisons have been performed to identify the most convenient and effective strategy when analysing these invariant objects and their stability.

Some results will be presented for both the circular and the elliptical restricted three body problems.

Arnold Diffusion via Scattering maps: A geometrical mechanism to detect global instability

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joint work with Amadeu Delshams (UPC) and Albert Granados

Abstract

We proved, in [1, 2], that for any non-trivial perturbation depending on any two independent harmonics of a pendulum and a rotor there is global instability, also called Arnold diffusion. The proof is based on the geometrical method and relies on the concrete computation of several scattering maps. A complete description of the different kinds of scattering maps taking place as well as the existence of piecewise smooth global scattering maps is also provided. Similar results apply for any non-trivial perturbation depending on any three independent harmonics of a pendulum and a 2 d.o.f rotor [3].

[1] Delshams A., Schaefer R.G., *Arnold diffusion for a complete family of perturbations*, Regular and Chaotic Dynamics 22 (2017), no. 1, 78–108.

[2] Delshams A., Schaefer R.G., *Arnold diffusion for a complete family of perturbations with two independent harmonics*, Discrete and Continuous Dynamical Systems 38 (2018), no. 12.

[3] Delshams A., Granados A., Schaefer R.G., *Arnold Diffusion for a Hamiltonian system with 3+1/2 degrees of freedom*, in preparation (2023).

A geometric method for computer assisted proofs in delay differential equations

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Abstract

A covering relation is a tool to express a concept that a given map $f : \mathbb{R}^n \to \mathbb{R}^n$ stretches in a proper fashion one set over another. Covering relations can be used to obtain coding for the orbits of the system, which is generally referred to as symbolic dynamics. Due to their geometric nature and open conditions, covering relations can be rigorously checked with computer assistance. We will present one possible exten- sion of the finite-dimensional covering relations to infinite dimensional systems, using compactness of the map.

A recently developed high-order Lohner-type rigorous algorithm [1] can be used to get enclosures of solutions to systems of Delay Differential Equations (DDEs) of quality good enough for various computer assisted proofs. We apply this method to verify covering relations for some Poincaré maps in the (subspace) of the phase space $C^0\left([-\tau, 0], \mathbb{R}^d\right)$ of DDEs to prove several unstable periodic orbits to Mackey-Glass equation in the chaotic regime of parameters, and the persistence of symbolic dynamics (semiconjugacy to a subshift on two symbols) in a chaotic ODE perturbed with a delayed term.

[1] Szczelina R., Zgliczyński P., *High-order Lohner-type algorithm for rigorous computation of Poincaré maps in systems of Delay Differential Equations with several delays*, Foundations of Computational Mathematics, in press (2023).

The maths of a photo induced hydrogel swimming robot: nonsmooth forcing dynamics

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Abstract

Certain hydrogel is light sensitive, since light provides heat and hydrogel shrinks under high temperature thus making the hydrogel photosensitive. When light shines on the hydrogel, the hydrogel shrinks and as light propagates through the hydrogel shrinks less thus bending towards the light. This hydrogel beam undergoes a vibration, governed by a wave PDE with a second derivative in time and 4th derivative in space coupled with a diffusion equation, a result of a competition between the elasticity of the beam and the photo induced bending. This photo induced vibration enables the hydrogel beam to swim in water. Based on dimensional energy analysis, we determine the stable vibration amplitude and construct phase diagrams for the increase and decrease of the oscillation amplitude, which are further confirmed experimentally. It is found that resonance can occur and damping plays an important role in determining the conditions for resonance. A mass-spring-damper ODE system subjected to a displacement dependent excitation force is developed to investigate the features in generalized self-excited oscillating systems. The prototypical PDEs can be well understood by the above simplified ODE model. This work lays a solid foundation for understanding self-excited oscillation and provides design guidelines for self-sustainable soft robots. It also puts forth another interesting question of whether chaos is involved in future work.

Shadowing of non-transversal heteroclinic chains

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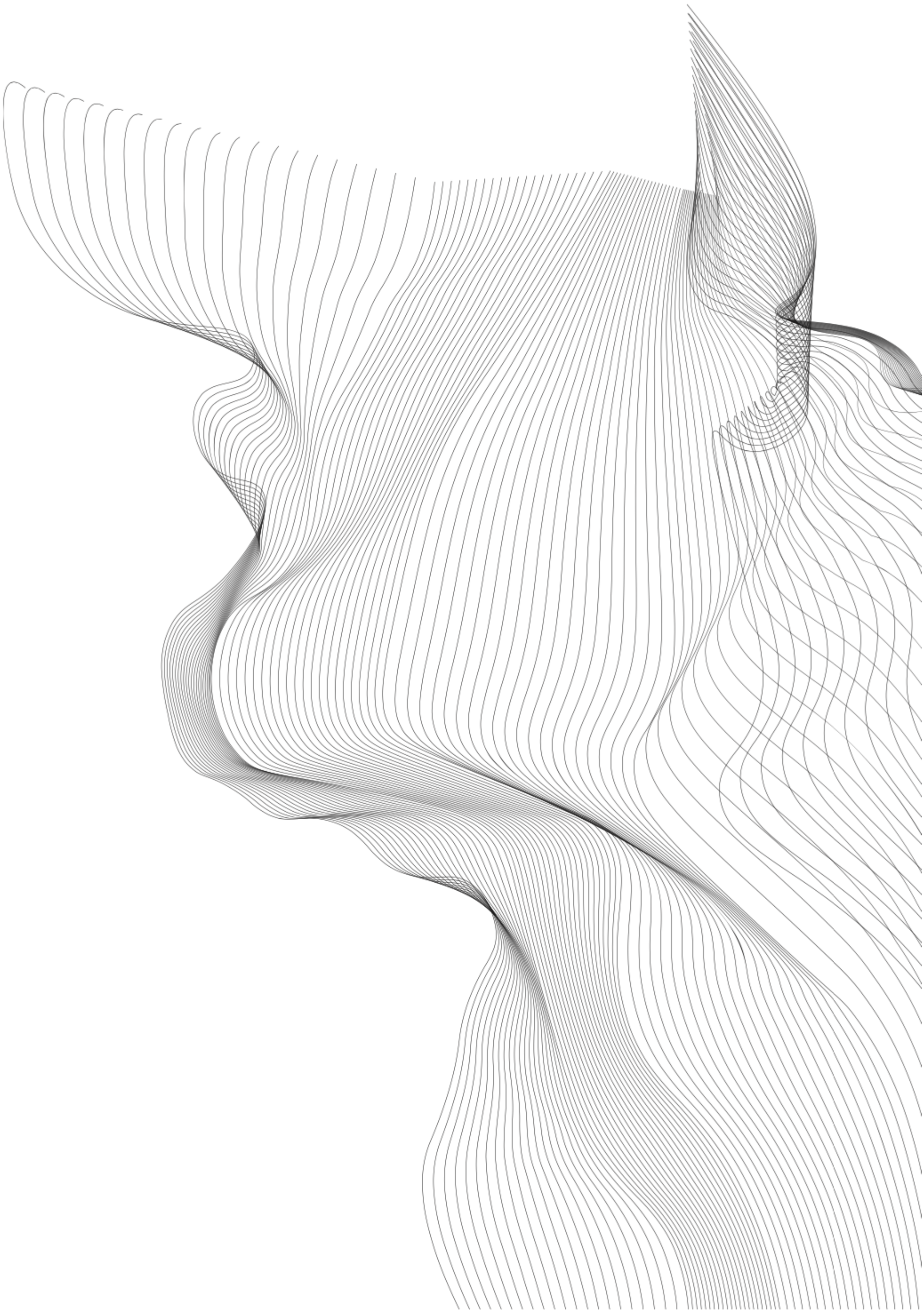
joint work with Amadeu Delshams

Abstract

We discuss a geometric method for the shadowing of nontransversal chain of heteroclinic connections based on the idea of dropping dimensions.

In our picture we think of evolving a disk of dimension k along a heteroclinic chain and when a given transition is not transversal, then we 'drop' one or more dimensions of our disk, i.e., we select a subdisk of lower dimension "parallel to expanding directions in future transitions". After at most k transitions, our disk is a single point and we cannot continue further. We will refer to this phenomenon as the *dropping dimensions* mechanism.

We illustrate this new mechanism for a generalization of toy model systems introduced by Colliander and all, Guardia and Kaloshin, Guardia-Hauss-Processi in the study of the energy transfer to high frequencies in the cubic defocusing nonlinear Schrödinger equation.



Thematic section

GCGT

Geometric and Combinatorial Group Theory

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Montserrat Casals Ruiz (Euskal Herriko Unibertsitatea & Ikerbasque, Bilbao)

Damian Osajda (Uniwersytet Wrocławski)

Enric Ventura (Universitat Politécnica de Catalunya & BSGMath, Barcelona)

SCHEDULE OF THE SECTION
Geometric and Combinatorial Group Theory

- Monday – September 4th

  16:00–17:00 Joan Porti, *A local approach to Anosov groups*

  coffee break

  17:30–18:15 Michał Marcinkowski, *The* L^1*-metric on* $Diff_0(M, area)$

  18:15–19:00 Karol Duda, *Torsion subgroups of small cancellation group*

- Tuesday – September 5th

  14:30–15:15 Ilya Kazachkov, *On the elementary theory of graph products of groups*

  15:15–16:00 Alexander Zakharov, *Relative order, spectrum of a subgroup and related algorithmic problems in groups*

  coffee break

  16:30–17:15 Oleg Bogopolski, *Exponential equations in acylindrically hyperbolic groups*

  17:15–18:00 Sangrok Oh, *Large-scale geometry of graph* 2*-braid groups*

- Wednesday – September 6th

  12:30–13:15 Piotr Nowak, *Coboundary expanders and Gromov hyperbolicity*

- Thursday – September 7th

  14:00–14:45 Yago Antolín, *Even Artin groups of FC-type*

  15:00–15:45 Motiejus Valiunas, *Biautomatic groups and non-positive curvature*

  coffee break

  16:30–17:15 Jacek Świątkowski, *Trees of graphs as boundaries of hyperbolic groups*

  17:15–18:00 Oihana Garaialde Ocañ, *Hausdorff dimension in profinite groups*

Even Artin groups of FC-type

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joint work with Islam Foniqi

Abstract

In this talk we will review some recent developments on a class of Artin groups that contains and shares many properties with right-angled Artin groups. We will concentrate on parabolic subgroups, and a version of a Tits Alternative for these families of groups.

Exponential equations in acylindrically hyperbolic groups

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joint work with Agnieszka Bier

Abstract

Let $\mathbf{X} = \{x_1, x_2, \dots\}$ be an infinite countable set. An *exponential equation* over a group G is an equation of the form

$$a_1 g_1^{x_{t_1}} a_2 g_2^{x_{t_2}} \dots a_n g_n^{x_{t_n}} = 1, \tag{B}$$

where $a_1, g_1, \dots, a_n, g_n$ are elements from G (called *coefficients*) and $x_{t_1}, \dots, x_{t_n} \in \mathbf{X}$ are *variables* (which take values in $\mathbb{Z}$).

We show that if G acts acylindrically on a hyperbolic space, then (B) is equivalent to a finite disjunction of finite systems of easier exponential equations which we call *elementarily loxodromic* and *elliptic*.

In the case where G is hyperbolic relative to a collection of peripheral subgroups $\{H_\lambda\}_{\lambda\in\Lambda}$ equation (B) is equivalent to a finite disjunction of finite systems of equations where each equation is *elementarily loxodromic*, *peripheral*, or *finitary*. As corollary, we prove in this case that the solution set of any exponential equation over G is $\mathbb{Z}$-semilinear if and only if the solution set of any exponential equation over every H_λ, $\lambda \in \Lambda$, is $\mathbb{Z}$-semilinear. This generalizes the result of Lohrey [1] that the $\mathbb{N}$-solution set of any exponential equation over a hyperbolic group is semilinear. Note that $\mathbb{Z}$-semilinearity is closely related to definability in the weak Presburger arithmetic.

A simple version of another our result says that if G is a finitely generated acylindrically hyperbolic group and Y is any finite generating set of G, then there exists a number $C > 0$ such that for any exponential equation (B) with generalized loxodromic $g_1, \dots, g_n$ and variables $x_1, \dots, x_n$, if this equation has a solution, then there exists a solution $(k_1, \dots, k_n)$ such that

$$\max_{i=1,\dots,n} |k_i| \leqslant \Big(\sum_{i=1}^{n} |a_i|_Y + \sum_{i=1}^{n} |g_i|_Y\Big) \cdot Cn.$$

This result generalizes and improves the main result in [2] where exponential equations over hyperbolic groups were considered. Finally, we introduce exponential-elementary and exponential-existential theories of groups.

[1] Lohrey M., *Knapsack in hyperbolic groups*, Journal of Algebra 545 (2020), no 1, 390–415.

[2] Myasnikov A., Nikolaev A., Ushakov A., *Knapsack problems in groups*, Mathematics of Computations 84 (2015), no. 292, 987–1016.

Hypercubical groups: from RAAGs to knot theory and beyond

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Abstract

Right-angled Artin groups form an outstanding and broadly studied class of groups due to their remarkable group-theoretic, cohomological, geometric, and combinatorial properties. To each RAAG one can associate a non-positively curved cubical complex, namely the Salvetti complex, whose universal covering is a CAT(0) cube complex and therefore it is contractible. Mimicking this phenomenon, we define the class of hypercubical groups as the class of those finitely generated groups G for which a certain cubical complex, namely the hypercubical complex of G, is contractible. The aim of this talk is to introduce such family of groups and show different cases where this phenomenon shows up. In particular, we will focus on a generalization of RAAGs and on a family of groups generalizing the link group of the Borromean rings.

Torsion subgroups of small cancellation group

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Abstract

We prove that torsion subgroups of groups defined by, $C(6)$, $C(4)$-$T(4)$ or $C(3)$-$T(6)$ small cancellation presentations are finite. This follows from more general results about locally elliptic action on small cancellation complexes.

[1] Duda K., *Torsion subgroups of small cancellation groups*, arXiv:2112.01912 (2021).

On the elementary theory of graph products of groups

Ilya Kazachkov

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Abstract

When studying the model theory of groups, it is natural to ask which group-theoretic constructions preserve the elementary theory. In 1959, Feferman and Vaught studied the first-order properties of direct products and showed, in particular, that the direct products of elementarily equivalent groups are elementarily equivalent. In contrast, invariance of the elementary equivalence for free products of groups was a long-standing conjecture which was recently solved by Sela (2017).

In this talk, we will first address the converse question: given two elementary equivalent free products of groups (or more generally, graph product of groups), when are the factors elementarily equivalent? We discuss some sufficient conditions and use our results to describe finitely generated groups elementarily equivalent to RAAGs whose underlying graph is a transitive forest.

The L^1-metric on $Diff_0(M, area)$

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joint work with M. Brandenbursky and E. Shelukhin

Abstract

Let M be a compact Riemannian manifold. There are a number of interesting metrics on the group of volume preserving diffeomorphisms of M, among them the L^1-metric. If M is an $(n > 2)$-dimensional disc, then the diameter of $Diff_0(M, vol)$ with the L^1-metric is finite by the celebrated result of A. Shnirelman. In the 2-dimensional case the situation is very different. In this talk I will show how to use braids to estimate the L^1-metric on $Diff_0(M, area)$ where M is a compact surface. As an application we construct many L^1-Lipschitz quasimorphisms on $Diff_0(M, area)$ and show that all right-angled Artin groups embed quasi-isometrically into $Diff_0(M, area)$.

Coboundary expanders and Gromov hyperbolicity

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joint work with Dawid Kielak

Abstract

I will discuss the concept of higher-dimensional expanders as a generalization of expander graphs. I will show that for a tower of residual finite coverings of a compact manifold, coboundary expansion in a certain dimension forces the fundamental group to be hyperbolic.

Hausdorff dimension in profinite groups

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Abstract

Hausdorff dimension is a tool to measure the size of fractal objects and subsets of a metric space. In [1], Y. Barnea and A. Shalev studied this notion in the context of profinite groups which are inverse limits of finite groups. In the first part of this talk, we explain some of the open questions and main results in this area (compare [1] and [2]). If time permits, we will explore a new result for finitely generated regular branch groups.

[1] Barnea Y., Shale A., *Hausdorff dimension, pro-p groups, and Kac-Moody algebras*, Transactions of the American Mathematical Society 349 (1997), no. 12, 5073–5091.

[2] Klopsch B., Thillaisundaram A., Zugadi-Reizabal A., *Hausdorff dimensions in p-adic analytic groups*, Israel Journal of Mathematics 231 (2019), 1–23.

[3] Shalev A., *Lie methods in the theory of pro-p groups, In New horizons in pro-p groups*, Birkhäuser Boston, Boston, 2000.

Large-scale geometry of graph 2-braid groups

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Abstract

Graph n-braid groups or n-braid groups over graphs are the fundamental groups of configuration spaces on graphs. Unlike configuration spaces on higher dimensional spaces, there is a discrete version of configuration space on a graph, which is a locally CAT(0) cube complex, and in particular, graph braid groups act geometrically on CAT(0) cube complexes. In this talk, using quasi-isometry invariants of CAT(0) square complexes, called intersection complexes, we will talk about the quasi-isometric classification of 2-braid groups over a special kind of graphs.

A local approach to Anosov groups

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joint work with M. Kapovich and B. Leeb

Abstract

Given finitely many elements of a semi-simple Lie group G of non compact type, I give a sufficient criterion so that they span an Anosov group (in particular discrete and word hyperbolic). The criterion is based on the action on the symmetric space G/K. As an application, I discuss the algorithmic recognizability of Anosov groups.

Trees of graphs as boundaries of hyperbolic groups

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Abstract

Despite many years of extensive study, various basic aspects of hyperbolic groups are still far from being well understood. For example, surprisingly little is still known about topological classification of Gromov boundaries of hyperbolic groups. We present here a new result in this direction (joint with Nima Hoda from Cornell University), which can be viewed as a complete classification of connected Gromov boundaries of the simplest possible form in topological dimension 1 (other that the cirle S^1). The corresponding spaces are called *regular trees of 2-connected graphs*, and they form a class of spaces which have particularly many local cut points (so that the Sierpinski curve and the Menger curve are beyond this class). We characterize completely, in algebraic terms, the family of all those hyperbolic groups G whose Gromov boundary ∂G is in this class. The latter characterization is obtained in terms of some features of the so called canonical JSJ splitting of G (along its 2-ended subgroups).

As a matter of fact, we indicate also a subclass in the above mentioned class of trees of graphs, called *trees of* Θ*-graphs*, which consists of even simpler spaces. We characterize also those hyperbolic groups G whose Gromov boundaries ∂G belong to this subclass. This exhibits some gradation of complexity among 1-dimensional connected Gromov boundaries ∂G, as well as among the corresponding groups G (in the latter case this gradation has a rather transparent algebraic flavour).

In order to understand the precise statements of our results, recall that the factors of the canonical JSJ splitting of a hyperbolic group fall into two separate classes. The well understood class of factors is that of so called *flexible factors*, and the more complex one is that of so called *rigid factors*. As we explain, rigid factors can be either isolated from each other (in some precise sense) be means of flexible factors, or they can form some groups of non-isolated rigid factors that we call *rigid cluster factors*. Recall also that *trees of graphs* form a natural class of topological spaces larger than that of regular trees of 2-connected graphs, and that a graph is 2-connected if it is connected and contains no cut vertex. A *thick* Θ*-graph* is a graph having precisely 2 vertices and at least 3 edges each of

which connects those two vertices. Recall finally that a hyperbolic group G is virtually Fuchsian iff its Gromov boundary ∂G is homeomorphic to the circle S^1. Our main results read then as follows.

Theorem A. *Let G be a 1-ended hyperbolic group which is not virtually Fuchsian. Then the following conditions are equivalent:*

(1) *∂G is homeomorphic to a tree of graphs;*
(2) *∂G is homeomorphic to a regular tree of 2-connected graphs;*
(3) *each rigid cluster factor of G is virtually free.*

Theorem B. *Let G be a 1-ended hyperbolic group that is not virtually Fuchsian. Then the following conditions are equivelent:*

(1) *∂G is homeomorphic to a tree of thick Θ-graphs;*
(2) *∂G is homeomorphic to a regular tree of thick Θ-graphs;*
(3) *G has no rigid factor in its canonical JSJ splitting.*

Biautomatic groups and non-positive curvature

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joint work with Sam Hughes

Abstract

Biautomatic groups, introduced in the early 1990s, arose as groups explaining formal language-theoretic aspects of word-hyperbolic groups. Many classes of non-positively curved finitely generated groups, such as hyperbolic, virtually abelian, cocompactly cubulated and Coxeter groups, are known to be biautomatic.

In this talk, I will give a brief introduction to the class of biautomatic groups and its relation to various classes of non-positively curved groups. Namely, I will outline some arguments showing that certain CAT(0) groups are not subgroups of biautomatic groups, that certain hierarchically hyperbolic groups are not biautomatic, and/or that the latter groups satisfy a weaker property of being *asynchronously* automatic.

Relative order, spectrum of a subgroup and related algorithmic problems in groups

Alexander Zakharov

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joint work with Jordi Delgado and Enric Ventura

Abstract

I will first survey some results about algorithmic problems for subgroups, such as subgroup membership problem. Then I will focus on the notions of relative order and spectrum of subgroups, and related algorithmic problems. The relative order of an element with respect to a subgroup is the minimal positive power in which the element belongs to the subgroup (and 0 if such a power does not exist), and the spectrum of a subgroup is the set of all possible relative orders with respect to that subgroup. I will present both positive and negative results on the computability of spectrum in different groups.

[1] Delgado J., Ventura E., Zakharov A., *Relative order and spectrum in free and related groups*, to appear in Communications in Contemporary Mathematics

Thematic section

GTM

Geometry and Topology of Manifolds

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Antonio Viruel (Universidad de Malaga)

Robert Wolak (Uniwersytet Jagielloński, Kraków)

SCHEDULE OF THE SECTION
Geometry and Topology of Manifolds

- Monday – September 4th

  16:00–17:00 Antonio Viruel, *Finite sets containing zero are mapping degree sets*

  coffee break

  17:30–18:15 Maciej Borodzik, *Concordance implies regular homotopy in codimension 2*

  18:15–19:00 Łukasz Michalak, *Reeb graph invariants of Morse functions, manifolds and groups*

- Tuesday – September 5th

  14:30–15:15 Aniceto Murillo, *The rational homotopy type of classifying spaces of homotopy automorphisms*

  15:15–16:00 Anna Gąsior, *Spin-structures on real Bott manifolds with Kaehler structure*

  coffee break

  16:30–17:15 Aleksandra Borówka, *Quaternionic manifolds with rotating circle action*

  17:15–18:00 Pawel Raźny, *A spectral sequence for free isometric lie algebra actions*

- Wednesday – September 6th

  12:00–12:30 Maciej Czarnecki, *Boundary of Hadamard foliations and laminations*

  12:30–13:00 Kacper Grzelakowski, *Triple points on Calabi-Yau threefolds*

  13:00–13:30 Wacław Marzantowicz, *Lefschetz number of equivariant mapping defined in equivariant cohomology theory*

- Thursday – September 7th

  14:30–15:15 Jordi Daura Serrano, *Large Finite group actions on aspherical manifolds*

  15:15–16:00 Martín Saralegui Aranguren, *Some Gysin sequences*

  coffee break

  16:30–17:15 Rafał Lutowski, *Complex Vasquez invariant*

  17:15–18:00 Andreas Zastrow, *The configuration spaces of the Earring Space are aspherical*

Concordance implies regular homotopy in codimension 2

Maciej Borodzik

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joint work with Mark Powell and Peter Teichner

Abstract

We introduce immersed Morse theory to prove that concordance implies regular homotopy in codimension 2.

Quaternionic manifolds with rotating circle action

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Abstract

B. Feix [3] (and D. Kaledin [5] independently) showed that there exists a hyperkähler metric on a neighbourhood of the zero section of the cotangent bundle of any real-analytic Kähler manifold. B. Feix provided an explicit construction of its twistor space and showed that any hyperkähler manifold admitting a rotating circle action near its maximal fixed point set arises locally in this way. The construction have been further generalized to hypercomplex manifolds (see Feix [4], Kaledin [6]), quaternionic manifolds (see Borówka, Calderbank [2]) and quaternion-Kähler manifolds (see Borówka [2]). In this talk we will discuss the cases of the construction. Then we will show how to apply it, to obtain a local classification result for quaternionic manifolds with rotating circle action near maximal fixed point set. Finally we will mention connections with c-map.

[1] Borówka A., *Quaternion-Kähler manifolds near maximal fixed point sets of S^1-symmetries*, AMPA 2020

[2] Borówka A., Calderbank D., *Projective geometry and the quaternionic Feix–Kaledin construction*, Transactions of the American Mathematical Society 2019.

[3] Feix B., *Hyperkahler metrics on cotangent bundles*, Journal für die reine und angewandte Mathematik 532 (2001), 33–46.

[4] Feix B., *Hypercomplex manifolds and hyperholomorphic bundles*, Mathematical Proceedings of Cambridge Philosophical Society 133 (2002), 443–457.

[5] Kaledin D., *Hyperkähler metrics on total spaces of cotangent bundles*, in D. Kaledin, M. Verbitsky, Hyperkähler manifolds, Math. Phys. Series **12**, International Press, Cambridge MA, 1999.

[6] Kaledin D., *A canonical hyperkähler metric on the total space of a cotangent bundle*, in Proceeedings of the Second Quaternionic Meeting, Rome (1999), World Scientific, Singapore, 2001.

Boundary of Hadamard foliations and laminations

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Abstract

An Hadamard foliation (or lamination) is a foliation (resp. lamination) of an Hadamard manifold (resp. Hadamard metric space) with all the leaves possesing this property i.e. being connected, simply connected, complete and nonopositively curved.

We shall discuss conditions under which ideal boundaries of leaves laminate the ideal boundary of carrying space and observe how does it work in case of the contracting boundary.

[1] Charney R., Sultan H., *Contracting boundaries of CAT(0) spaces*, Journal of Topology 8 (2015), 93–117.

[2] Czarnecki M., *Hadamard foliations of H^n*, Differential Geometry and its Applications 20 (2004), 357–365.

[3] Czarnecki M., *Umbilical routes along geodesics and hypercycles in the hyperbolic space*, Differential Geometry and its Applications 64 (2019), 47–58.

Large finite group actions on aspherical manifolds

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Abstract

The theory of finite transformation groups studies the symmetries of objects like topological spaces or manifolds by means of finite group actions. Two fundamental question are the following: Given a closed manifold, which finite groups can act effectively on it? Conversely, which topological properties should a closed manifold M have if we know a collection of finite groups actions on M? The answer to these questions in full generality is currently out of reach. One way to simplify them is to study which properties do large finite groups acting on M should fulfil.

In this talk we will show how we can address these questions by studying the Jordan property on the homeomorphism group of closed manifolds or by introducing invariants like the discrete degree of symmetry (see [1] for a recent survey on the topic). We will focus on the case of aspherical manifolds, providing new examples of closed manifolds with Jordan homeomorphism group and computing their discrete degree of symmetry. We would also introduce a theory of iterated finite group actions, which will help us to study rigidity questions on nilmanifolds.

[1] Riera I.M., *Actions of large finite groups on manifolds*, arXiv:2303.07784 (2023).

Spin-structures on real Bott manifolds with Kaehler structure

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Abstract

The main purpose of the article is the problem of existence of spin structures on real Bott manifolds which admit a Kähler structure.

It is known that spin structures on finite quotients of flat tori (flat manifolds) is strictly connected to Sylow 2-subgroups of their holonomy groups. But even knowing that the problem isn't solved for flat manifolds with holonomy group being elementary abelian 2-group, although few years ago significant progress has been made in the case when the action of a finite group on a torus is in a sense diagonal. We take advantage of it and show that when a real Bott manifold admits a Kähler structure then existence of a spin structure on it can be formulated with an easy combinatorial condition.

Triple points on Calabi-Yau threefolds

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Abstract

We discuss the bounds for the number of ordinary triple points on complete intersection Calabi-Yau threefolds in projective spaces and for Calabi-Yau threefolds in weighted projective spaces. In particular we show that in $\mathbb{P}^5$ the intersection of a quadric and a quartic cannot have more than 10 ordinary triple points. We provide examples of complete intersection Calabi-Yau threefolds with multiple triple points. We obtain the exact bound for a sextic hypersurface in $\mathbb{P}[1:1:1:1:2]$ which is 10. We also discuss Calabi-Yau threefolds that cannot admit triple points.

[1] Cynk S., *Hodge numbers of hypersurfaces in $\mathbb{P}^4$ with ordinary triple points*, Advances in Geometry 21 (2021), no. 2, 293–298.

[2] Dolgachev I., *Weighted projective varieties*, Group actions and vector fields (1981), 34–71.

[3] Dolgachev I., *Corrado Segre and nodal cubic threefolds*, In: Casnati G., Conte A., Gatto L., Giacardi L., Marchisio M., Verra A. (eds) From Classical to Modern Algebraic Geometry. Trends in the History of Science. BirkhĂ¤user (2016), 429–450.

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[6] Fortuna E., Frigerio R., Pardini R., *Projective Geometry Solved Problems and Theory Review*, Springer International Publishing Switzerland, 2016.

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[11] Roberts J., *Hypersurfaces with Nonsingular Normalization and Their Double Loci*, Journal of Algebra 53 (1978), 253–267.

[12] Stevens J., *Sextic surfaces with ten triple points*, arXiv:0304060v1 (2003).

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Complex Vasquez invariant

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joint work with Anna Gąsior

Abstract

A flat manifold is a closed connected Riemannian manifold with vanishing sectional curvature. By the Auslander-Kurnanishi theorem, every finite group is a holonomy group of some flat manifold. In 1970 Vasquez showed, that for every finite group G there is a natural number $n(G)$ such that every flat manifold X' with holonomy group G is a flat toral extension of a flat manifold X of dimension less than or equal to $n(G)$. In particular this means that we have a fiber bundle

$$T \to X' \to X,$$

where T is a flat torus.

In the talk I will present an analogue of Vasquez number, which is defined for the family compact flat Kähler manifolds. Besides showing some dependencies between real and complex versions of the invariant, I will focus on the problem of the projection map of the bundle to be holomorphic.

Lefschetz number of equivariant mapping defined in equivariant cohomology theory

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joint work with Arturo Espinosa-Baro

Abstract

In late 80-ties of twenty century in the works of M. Atiyah and G. Segal, and later of F. Hirzebruch and T. Höfer, an invariant related to the Euler characteristic of the orbit space had been studied. This invariant was inspired by a paper of the theoretical physicists L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten. In the first two articles it was shown that this invariant is expressed in the terms of equivariant K-theory. Independently, a decade earlier the second author studied a Lefschetz number $\lambda_G(f)$ of equivariant map $f : X \overset{G}{\to} X$ in the K_G^*-theory showing its main properties. The aim of this project is to show that: The Lefschetz number $\lambda_G(f) \in \mathbb{R}(G) \otimes \mathbb{C}$ shares majority of properties of the Euler characteristic type invariant mentioned above generalizing the latter. Its specification for $f = \mathrm{id}_{|X}$ in K_G^*-theory gives previously defined invariant thus extends results of referred works. Applications to study the existence of fixed orbits of maps equivariant with respect of co-finite groups, e.g. the crystallographic group,s is in progress.

Reeb graph invariants of Morse functions, manifolds and groups

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Abstract

The Reeb graph of a Morse function on a closed manifold is obtained by contracting each connected component of its level sets. There are two necessary and sufficient conditions for a finite graph to be realized as the Reeb graph of a Morse function on a given closed manifold: it needs to have the so-called good orientation and its first Betti number cannot exceed the corank of the fundamental group of the manifold. Moreover, any free quotient of this group can be represented as the Reeb epimorphism of a Morse function which is induced on fundamental groups by the quotient map from the manifold to the Reeb graph. It leads to the study of relations between the notions of equivalence of epimorphisms onto free groups, cobordism of systems of hypersurfaces and topological conjugation of Morse functions.

However, the realization of a graph as the Reeb graph of a Morse function is possible only up to a homeomorphism of graphs in general. The minimum number of degree 2 vertices in Reeb graphs of Morse functions is a strong invariant of the topology of the manifold. It has three essentially different lower bounds, which for orientable 3-manifolds are improved by the inequality involving the Heegaard genus. Moreover, another bound is defined in terms of finite presentations of the fundamental group. We use Freiheitssatz, a fundamental fact from one-relator groups, to calculate it in some cases.

The rational homotopy type of classifying spaces of homotopy automorphisms

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joint work with Mario Fuentes and Yves Félix and extracted from [1, 2]

Abstract

We will describe the rational homotopy type of classifying spaces of homotopy automorphisms of nilpotent complexes in terms of certain Lie algebras of derivations.

[1] Fuentes M., Félix Y., Murillo A., *Lie models of homotopy automorphisms monoids and classifying fibrations*, Advances in Mathematics 402 (2022), 1–64.

[2] Fuentes M., Félix Y., Murillo A., *Realization of Lie algebras of derivations and moduli spaces of some rational homotopy types*, arXiv:2206.14124v1 (2022), to appear in Algebraic and Geometric Topology.

A spectral sequence for free isometric lie algebra actions

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Abstract

Assume that (M, g) is a compact Riemannian manifold with a free isometric action of a Lie algebra $\mathfrak{g}$. We present a new spectral sequence, arising from the restriction of the standard filtration of the spectral sequence of the foliation $\mathcal{F}_G$ (see [1]) by orbits of the $\mathfrak{g}$-action to a certain subcomplex of the de Rham complex, which connects the basic cohomology of the foliation, the Lie algebra cohomology of $\mathfrak{g}$ and the de Rham cohomology of M. The construction is a generalization of the Gysin long exact sequence in Sasakian Geometry (see [3]) and is an extension of our prior work [4] on $\mathcal{K}$-structures (see [2]).

[1] Álvarez López J.A., *A finiteness theorem for the spectral sequence of a Riemannian foliation*, Illinois Journal of Mathematics 33 (1989), no. 1, 79–92.

[2] Blair D.E., *Geometry of manifolds with structural group $\mathcal{U}(n) \times \mathcal{O}(s)$*, Journal of Differential Geometry 4 (1970), no. 2, 155–167.

[3] Boyer C.P., Galicki K., *Sasakian Geometry*, Oxford Mathematical Monographs, Oxford University Press, 2007.

[4] Raźny P., *Cohomology of manifolds with structure group $U(n) \times O(s)$*, Geometriae Dedicata 217 (2023), no. 58.

Some Gysin sequences

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Abstract

We are studying a smooth isometric action $\Phi\colon G \times M \to M$ of a Lie group on a manifold M, and our goal is to establish certain connections between the cohomology of M and that of the quotient space M/G.

Which cohomology groups should we consider for M/G?

In the case of an almost free action, the basic cohomology of M/G is sufficient. However, when more complex isotropy subgroups are involved, the basic intersection cohomology of M/G becomes a better adapted tool. In addition to introducing this cohomology, we will conclude the talk by presenting several Gysin sequences that cover specific cases such as $G = \mathbb{R}$, $\mathbb{S}^1$, and $\mathbb{S}^3$.

[1] Royo Prieto J.I., Saralegi Aranguren M., *The Gysin sequence for $\mathbb{S}^3$-actions on manifolds*, Publicationes Mathematicae Debrecen 83 (2013) no. 3, 275–289.

[2] Royo Prieto J.I., Saralegi Aranguren M., *The Gysin braid for S^3-actions on manifolds*, arXiv:2301.09002 (2023).

Finite sets containing zero are mapping degree sets

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joint work with Cristina Costoya and Vicente Muñoz

Abstract

In this lecture, we address several questions which have been raised about $D(M, N)$, the set of mapping degrees between two oriented closed connected manifolds M and N of the same dimension:

$$D(M, N) = \{d \in \mathbb{Z} \mid \exists f : M \to N, \deg(f) = d\}.$$

Neofytidis-Wang-Wang [1, Problem 1.1] discuss the problem of finding, for any set $A \subset \mathbb{Z}$ containing 0, two oriented closed connected manifolds M and N of the same dimension such that $A = D(M, N)$. Note that $0 \in A$ is a necessary condition as the constant map $M \to N$ is of degree zero.

A cardinality argument shows that when A is an infinite set, the problem above is solved in the negative [1, Theorem 1.3].

In contrast, we shall show that given A, any finite set of integers containing 0, there exist (infinitely many) oriented closed connected manifolds M, N such that $A = D(M, N)$. Moreover, the manifolds M, N above can be chosen to be either 3-dimensional, $(4m - 1)$-dimensional for $m \geqslant 4$ or simply connected.

[1] Neofytidis C., Wang S., Wang Z., *Realising sets of integers as mapping degree sets*, to appear in Bulletin of London Mathematical Society.

The configuration spaces of the Earring Space are aspherical

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Abstract

The Earring Space (that during the past 40 years has usually been called “the Hawaiian Earrings”) is a subspace of the plane which very much resembles a graph, apart from having not the topology of a graph at one of its points. But due to the accumulation at that point it must be considered as a non-triangulable space. It is known to be aspherical as a one-dimensional space [3, Corl.], a planar space [6, 2], and by having a generalized universal contractible covering [5, Expl.4.15+(U_3)]. Conversely graphs, open subsets of the plane or any surface different from S^1 and $\mathbb{R}P^2$ are known to have also aspherical (ordered and unordered) configuration spaces [4, Corl.2.2], [1, Corl.3.4]. The fact that it is not known whether these results extend to the nearest non-triangulable spaces has been brought to my attention by Daciberg Lima Gonçalves. While it is at the time of writing this abstract it is not clear, whether generalized covering space theory and the action of their deck-transformation groups in this case suffice to extend the classical proofs also in the case of the Earring Space, the talk will describe a more direct proof that the ordered and unordered configuration spaces of the Earring Space are, indeed, aspherical.

[1] Abrams A.D., *Configuration spaces of colored graphs*, Geometriae Dedicata 92 (2002), 185–194.

[2] Cannon J.W., Conner G.R., Zastrow A., *One-dimensional sets and planar sets are aspherical*, Topology and its Applications 120 (2002), no. 1-2, 23–45.

[3] Curtis M.L., Fort M.K., *Homotopy groups of one-dimensional spaces*, Proceedings of the American Mathematical Society 8 (1957), 577–579.

[4] Fadell E., Neuwirth L., *Configuration spaces*, Mathematica Scandinavica 10 (1962), 111–118.

[5] Fischer H., Zastrow A., *Generalized universal covering spaces and the shape group*, Fundamenta Mathematicae 197 (2007), 167–196.

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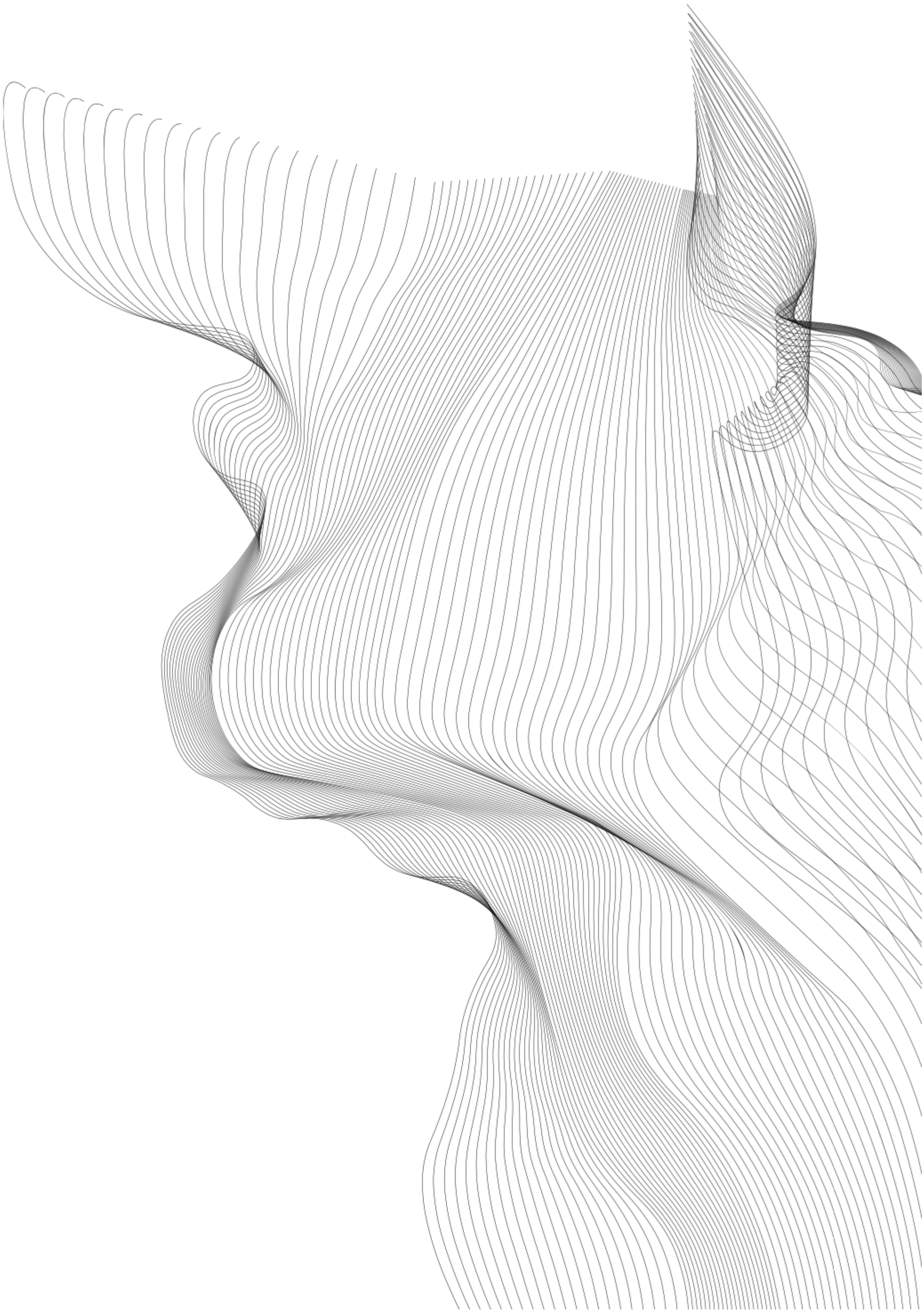

Thematic section

HSM
Hilbert Spaces Methods

ORGANIZERS:

Dariusz Cichoń (Uniwersytet Jagielloński, Kraków)

Jose Esteban Gale Gimeno (Universidad de Zaragoza)

Eva Gallardo Gutierrez (Universidad Complutense de Madrid)

Paweł Pietrzycki (Uniwersytet Jagielloński, Kraków)

Jan Stochel (Uniwersytet Jagielloński, Kraków)

Franciszek Hugo Szafraniec (Uniwersytet Jagielloński, Kraków)

SCHEDULE OF THE SECTION
Hilbert Spaces Methods

- Monday – September 4th

  16:00–16:30 Antonio Galbis, *Hausdorff Operators on Fock spaces*

  16:30–17:00 Pedro J. Miana, *Hilbertian Hardy-Sobolev spaces on a half-plane*

  coffee break

  17:30–18:00 Paweł Pietrzycki, *Arveson's hyperrigidity*

  18:00–18:30 Wojciech Młotkowski, *On freely quasi-infinitely divisible distributions*

  18:30–19:00 Łukasz Kosiński, *TBA*

- Tuesday – September 5th

  14:30–15:00 Miguel Monsalve-López, *Dunford property for composition operators on H^p-spaces*

  15:00–15:30 Daniel Seco, *Distribution of primes and approximation on weighted Dirichlet spaces*

  15:30–16:00 Wiktor Ejsmond, *A cyclic Fock space of type B*

  coffee break

  16:30–17:00 Marcin Marciniak, *Application of Generalized Gleason Theorem to semifinite factors*

  17:00–17:30 Sergiusz Kużel, *Dual frames and Naimark dilation theorem*

A cyclic Fock space of type B

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Abstract

We introduce a two-parameter function $\phi_{q,s}$ on the infinite hyperoctahedral group, which is a bivariate refinement of the reflection length keeping track of the long and the short reflections separately. We provide a complete characterization of the parameters q, s when the signed reflection function $\phi_{q,s}$ is positive definite and we prove that this condition holds if and only if $\phi_{q,s}$ is an extreme character of the infinite hyperoctahedral group. We construct the corresponding representations as a natural action of the hyperoctahedral group $B(n)$ on the tensor product of n copies of a vector space, which gives a two-parameter analog of the classical construction of Schur–Weyl.

We apply our characterization to construct a cyclic Fock space of type B which generalizes the one-parameter construction in type A found previously by Bożejko and Guta. We also construct a new cyclic Gaussian operator of type B and we relate its moments with the Askey–Wilson–Kerov distribution by using the notion of cycles on pair-partitions, which we introduce here.

Hausdorff Operators on Fock spaces

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Abstract

We discuss boundedness and compactness of Hausdorff operators acting on Fock spaces. The compactness result for Hausdorff operators on the Fock space F_α^∞ is extended to more general Banach spaces of entire functions with weighted sup norms defined in terms of a radial weight. We also include some results on p-summing operators.

[1] Blasco O., Galbis A., *Boundedness and compactness of Hausdorff operators on Fock spaces*, preprint.

[2] Bonet J., *Hausdorff operators on weighted Banach spaces of type H^∞*, Complex Analysis and Operator Theory 16 (2022), no.1.

[3] Galanopoulos P., Stylogiannis G., *Hausdorff operators on Fock Spaces and a coefficient multiplier problem*, Proceedings of the American Mathematical Society 151 (2023), no.7, 3023–3035.

[4] Tung J., *Taylor coefficients of functions in Fock spaces*, Journal of Mathematical Analysis and Applications 318 (2006), 397–409.

Dual frames and Naimark dilation theorem

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Abstract

Although Naimark dilation theorem was originally stated in 1940, it still finds many important applications in signal processing, computer science, engineering, quantum information theory. The original result, established by Naimark for the case of a generalized resolution of identity [3] was analogized for a Parseval frame by Han and Larson [1]:
Theorem 1. *Let $F_e = \{e_j,\ j \in \mathbb{N}\}$ be a Parseval frame in a Hilbert space K. Then there exists a Hilbert space M and a complementary Parseval frame $F_m = \{m_j,\ j \in \mathbb{N}\}$ in M such that the set of vectors $F_{e \oplus m} = \{e_j \oplus m_j,\ j \in \mathbb{N}\}$ is an orthonormal basis of $H = K \oplus M$.*

A complementary Parseval frame F_m was described in [1] through the identification of H with $l_2(\mathbb{N})$. Such a description of F_m is not always adequate. Since Theorem 1 holds numerous significant applications, it becomes crucial to discover a relatively simple representation of the complementary frame F_m using the original Parseval frame F_e. In this talk, a method for explicitly constructing of F_m in scenarios where the initial frame F_e includes a Riesz basis is presented. These findings are subsequently utilized in the construction of dual frames [2].

[1] Han D., Larson D.R., *Frames, bases and group representations*, Memoirs of the American Mathematical Society 147 (2000), no. 697.

[2] Kamuda A., Kużel S., *On description of dual frames*, Applied and Computational Harmonic Analysis 56 (2021), 351–367.

[3] Naimark M.A., *Spectral functions of a symmetric operator*, Izvestiya Rossiiskoi Akademii Nauk. SSSR Seriya Matematicheskaya 4 (1940), 271–318.

Application of Generalized Gleason Theorem to semifinite factors

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Abstract

The Generalized Gleason Theorem [1] states that for any von Neumann algebra M with no direct summand of type I_2 and any Banach space X each bounded X-valued measure μ on the projection lattice of M extends uniquely to a bounded linear operator from M to X. We consider the case when X is another von Neumann algebra N and μ is a positive measure. Then naturally μ extends to a positive map $\phi : M \to N$. We are focused on the following two problems:

- Can the assumptions be weakened?
- Is it possible to characterize the type of positivity of ϕ such as k-positivity, complete positivity, decomposability, etc. in terms of the measure μ?

It turns out that in both problems rank properties of the measure μ play crucial role. Motivated by the results of [2] and Wigner type theorem for semifinite factors [3] we provide some sufficient conditions on the measure μ for decomposability of the map ϕ.

[1] Bunce L.J., Wright D.M., *The Mackey-Gleason problem*, Bulletin of the American Mathematical Society 26 (1992), p. 288–293.

[2] Marciniak M., *On extremal positive maps between type I factors*, Banach Center Publications 89 (2010), p. 201–221.

[3] Qiana W., Wang L., Wu W., Yuan W., *Wigner-type theorem on transition probability preserving maps in semifinite factors*, Journal of Functional Analysis 276 (2019), p. 1773–1787.

Hilbertian Hardy-Sobolev spaces on a half-plane

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joint work with José E. Galé, Valentin Matache
and Luis Sánchez–Lajusticia

Abstract

In this talk we deal with a scale of reproducing kernel Hilbert spaces $H_2^{(n)}$, $n \geqslant 0$, which are linear subspaces of the classical Hilbertian Hardy space on the right-hand half-plane $\mathbb{C}^+$. They are obtained as ranges of the Laplace transform in extended versions of the Paley-Wiener theorem which involve absolutely continuous functions of higher degree. An explicit integral formula is given for the reproducing kernel $K_{z,n}$ of $H_2^{(n)}$, from which we can find the estimate $\|K_{z,n}\| \sim |z|^{-1/2}$ for $z \in \mathbb{C}^+$. Then composition operators $C_\varphi : H_2^{(n)} \to H_2^{(n)}$, $C_\varphi f = f \circ \varphi$, on these spaces are discussed, giving some necessary and some sufficient conditions for analytic maps $\varphi : \mathbb{C}^+ \to \mathbb{C}^+$ to induce bounded composition operators. These results are included in two joint papers with J.E. Galé, V. Matache and L. Sánchez Lajusticia.

Pedro J. Miana has been partially supported by Project: ID2019-105979GBI00, DGI-FEDER, of the MCEI and Project E48-20R, Gobierno de Aragón, Spain.

[1] Galé J.E., Matache V., Miana P.J., Sánchez-Lajusticia L., *Hilbertian Hardy-Sobolev spaces on a half-plane*, Journal of Mathematical Analysis and Applications 489 (2020), 124–131.

[2] Galé J.E., Miana P.J., Sánchez–Lajusticia L., *RKH spaces of Brownian type defined by Cesàro–Hardy operators*. Analysis and Mathematical Physics 11 (2021), no. 3, 1–34.

On freely quasi-infinitely divisible distributions

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Abstract

Inspired by the notion of quasi-infinite divisibility (QID), we study the class of freely quasi-infinitely divisible (FQID) distributions on $\mathbb{R}$, which admit the free Lévy-Khintchine-type representation with signed Lévy measure. We prove several properties of the FQID class, some of them in contrast to those of the QID class. For example, a FQID distribution may have negative Gaussian component, and the total mass of its signed Lévy measure may be negative. We provide a characteristic triplet, with the Lévy measure having nonzero negative part, which is at the same time classical and free characteristic triplet.

[1] Hotta I., Młotkowski W., Sakuma N., Ueda Y., *On freely quasi-infinitely divisible distributions*, arXiv:2107.09473.

[2] Lindner A., Pan L., Sato K., *On quasi-infinitely divisible distributions*, Transactions of the American Mathematical Society 370 (2018), no.12, 8483–8520.

Dunford property for composition operators on H^p-spaces

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joint work with E. A. Gallardo-Gutiérrez and F. J. González-Doña

Abstract

The Dunford property (C) for composition operators on H^p-spaces $(1 < p < \infty)$, as well as for their adjoints, is completely characterized within the class of those induced by linear fractional transformations of the unit disc. As a consequence, it is shown that the Dunford property is stable in such a class addressing a particular instance of a question posed by Laursen and Neumann.

Arveson's hyperrigidity

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joint work with Jan Stochel

Abstract

Motivated both by the fundamental role of the classical Choquet boundary in classical approximation theory, and by the importance of approximation in the contemporary theory of operator algebras, Arveson introduced hyperrigidity as a form of approximation that captures many important operator-algebraic phenomena. I will discuss new results on the notion of hyperrigidity such as the characterization of spectral measures, the intertwining theorem, the relationship between the convergence of subnormal operators in weak and strong operator topologies, and new examples of sets of generators that are hyperrigid.

[1] Pietrzycki P., Stochel J., *Subnormal nth roots of quasinormal operators are quasinormal*, Journal of Functional Analysis 280 (2021), no. 12.

[2] Pietrzycki P., Stochel J., *Two-moment characterization of spectral measures on the real line*, Canadian Journal of Mathematics 75 (2022), no. 4.

[3] Pietrzycki P., Stochel J., *On nth roots of bounded and unbounded quasinormal operators*, Annali di Matematica Pura ed Applicata 202 (2022), p. 1313-1333.

[4] Pietrzycki P., Stochel J., *Hyperrigidity: characterizations of spectral measures and convergence of subnormal operators*, in preparation.

Distribution of primes and approximation on weighted Dirichlet spaces

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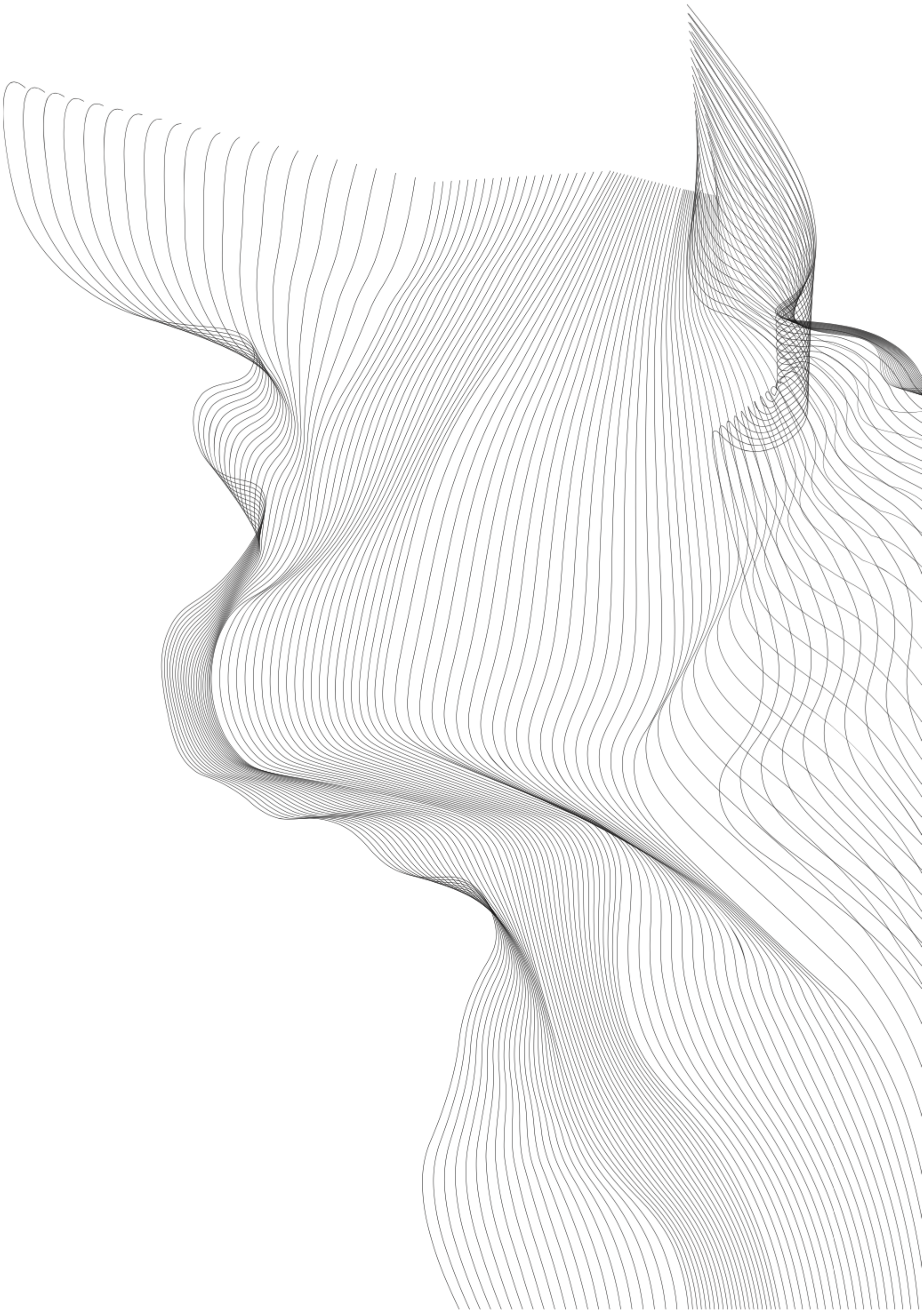
joint work with Eva A. Gallardo-Gutiérrez

Abstract

We study zero-free regions of the Riemann zeta function ζ related to an approximation problem in the weighted Dirichlet space D_{-2} which is known to be equivalent to the Riemann Hypothesis since the work of Báez-Duarte. We prove, indeed, that analogous approximation problems for the standard weighted Dirichlet spaces D_α when $\alpha \in (-3, -2)$ give conditions so that the half-plane $\{s \in C : \Re(s) > -\frac{\alpha+1}{2}\}$ is also zero-free for ζ. Moreover, we extend such results to a large family of weighted ℓ^p-spaces of analytic functions. As a particular instance, in the limit case $p = 1$ and $\alpha = -2$, we provide a new proof of the Prime Number Theorem.



Thematic section

ME
Mathematical Economics

ORGANIZERS:

Carlos Herves Beloso (Universidade de Vigo)

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Emma Moreno Garcia (Universidad de Salamanca)

Agnieszka Wiszniewska-Matyszkiel (Uniwersytet Warszawski)

SCHEDULE OF THE SECTION
Mathematical Economics

- Tuesday – September 5th

  14:30–15:00 Anna Jaśkiewicz, *Time-consistency in the mean-variance problem: a new perspective*

  15:00–15:30 Dariusz Zawisza, *Optimal portfolio selection on the bond market*

  15:30–16:00 Juan Pablo Rincon Zapatero, *TBA*

  coffee break

  16:30–17:00 Honorata Sosnowska, *Three voting methods diminishing manipulation tendency. Comparison of properties*

  17:00–17:30 Emma Moreno-García, *Economies with rights: efficiency and inequality*

  17:30–18:00 Carlos Hervés-Beloso, *Ordinar and cardinal preferences. Is there a single, canonical utility representation of cardinal preferences?*

  18:00–18:30 Lesław Gajek, *Ruin probabilities in regime - switching models with imprecise information about the switch*

- Thursday – September 7th

  14:30–15:00 Jakub Bielawski, *Chaos in learning dynamics*

  15:00–15:30 Dominika Machowska, *Closed-loop Nash equilibrium for a partial differential game with application to competitive personalized advertising*

  15:30–16:00 Agnieszka Wiszniewska-Matyszkiel, *A mathematical model of "the tragedy of the commons" with relation to counteracting pandemic*

  coffee break

  16:30–17:00 Łukasz Woźny, *A global version of Tarski-Kantorovitch theorem for correspondences*

  17:00–17:30 Agnieszka Lipieta, *Optimal demand-driven eco-mechanisms leading to equilibrium in competitive economy*

  17:30–18:00 Marta Kornafel, *An economic growth model with ecological investments*

Chaos in learning dynamics

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Abstract

We study behavior of agents in a congestion game when the agents make choices of strategies according to a learning algorithm. We consider machine learning algorithms (Multiplicative Weights Update, Follow the Regularized Leader) and a behavioral algorithm (Experience Weighted Attractions). We show that when the size of the population of agents is small all trajectories of the system converge to an equilibrium (Nash Equilibrium or Quantal Response Equilibrium). However, when the size of the population of agents increases the equilibrium loses stability and we observe periodic/chaotic behavior of the agents. Moreover, the size of the chaotic regime is smaller in the behavioral algorithm in comparison to machine learning algorithms.

Ruin probabilities in regime – switching models with imprecise information about the switch

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Abstract

Regime-switching Markovian models can describe natural disasters such as floods more accurately then the corresponding models without a switch. However, when using switching models, we must first provide them with a transition matrix that can be estimated from the data. In this lecture we will investigate how much the inaccuracy in determining the transition matrix of the regime-switching Sparre Andersen model can affect the infinite-horizon ruin probability of the insurer.

Time-consistency in the mean-variance problem: a new perspective

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joint work with N. Bäuerle [1]

Abstract

We investigate discrete-time mean-variance portfolio selection problems viewed as a Markov decision process. We transform the problems into a new model with deterministic transition function for which the Bellman optimality equation holds. In this way, we can solve the problem recursively and obtain a time-consistent solution, that is an optimal solution that meets the Bellman optimality principle. We apply our technique for solving explicitly a more general framework.

[1] Bäuerle N., Jaśkiewicz A., *Time-consistency in the mean-variance problem: a new perspective*, submitted (2023).

An economic growth model with ecological investments

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Abstract

We will present an economic growth model, which takes into account the investments in creating an infrastructure for production and distribution of renewable energy. In the economy the energy is supplied from both nonrenewable and renewable resources. The dynamics of the model in the context of stability of equilibria will be analysed and economic conclusions drawn.

Optimal demand-driven eco-mechanisms leading to equilibrium in competitive economy

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Abstract

We examine new mechanisms that introduce environmentally friendly eco-changes involving the elimination of noxious commodities and take into account the structure of demand without a detrimental effect to agents' position. In the era of the fourth industrial revolution, these mechanisms allow eliminating unnecessary services or goods that are being replaced by modern technologies. We define optimal mechanisms under the criterion of distance minimization, when a small number of detrimental commodities is excluded from production processes as well as when producers are change-averse. The results have the form of theorems with rigorous proofs.

[1] Denkowska A., Lipieta A., *Optimal demand-driven eco-mechanisms leading to equilibrium in competitive economy*, Central European Journal of Economic Modelling and Econometrics 14 (2022), 225–262.

[2] Lipieta A., Malawski A. *Price versus Quality Competition: In Search for Schumpeterian Evolution Mechanisms*, Journal of Evolutionary Economics 26 (2016), 1137–1171.

Closed-loop Nash equilibrium for a partial differential game with application to competitive personalized advertising

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Abstract

This paper is devoted to an N-person partial differential game whose dynamics of the state variable is described by a hyperbolic differential equation with certain boundary and initial conditions while the objective of each player is given by a finite horizon accumulated payoff functional with discounting. We extend the concept of a closed-loop Nash equilibrium for a partial differential game with the dynamics of the states described by a hyperbolic differential equation (a transport equation). We propose the definition of a dual closed-loop Nash equilibrium for which we give sufficient conditions. Moreover, we present the relationship between the Nash equilibria with the dual closed-loop and the classical closed-loop information structure. We apply the new results to the goodwill dynamics model in which the goodwill is influenced by personalized advertising and consumers' recommendations for which we construct a dual closed-loop Nash equilibrium and we examine its economic properties.

Economies with rights: efficiency and inequality

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joint work with Carlos Hervés-Beloso
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Abstract

Very large inequality levels originate negative externalities. This paper introduces, within a general equilibrium framework, tradable consumption rights to obtain efficient outcomes, reduce inequality, and improve social welfare. This mechanism would be easily implementable with the necessary support of the law.

* C. Hervés-Beloso and E. Moreno-García acknowledge the support of Research Grant PID2019-106281GB-I00 (Ministerio de Economía y Competitividad) and ECOBAS (Xunta de Galicia). F. Martínez Concha acknowledges the support of the grant ANID PIA/PUENTE AFB220003.

Three voting methods diminishing manipulation tendency. Comparison of properties

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Abstract

The small voting bodies, such as juries of classical music or sports competitions, are considered. In such bodies, very often, various types of rankings are used. Jury members may try to manipulate voting by giving high scores to their favorites and low scores to their opponents. Gibbard-Satherwaitte's, [1, 3], theorem shows that there is no voting method that preserves against manipulation, different from dictatorship. We can only find some methods which may diminish manipulation tendency. The most popular such methods are based on two methods, the Olympic Mean and winsorizing. Members of a jury give scores to the contestants.

The most known score system is the Borda Count. There are n contestants. Jurors give n points to the best, $n-1$ points to the second best, and so on till 1 point to the worst. For every contestant, the mean of jurorsâ€™ scores is computed. The contestant with the highest mean wins. The Borda Count is very sensitive to manipulation.

The Olympic Mean is a kind of trimmed mean. One lowest score and one highest score are removed, and the mean of remained scores is computed. The contestant with the highest such mean wins. This method and its generalizations were used during the International Henryk Wieniawski Violin Competitions, the main violin competition in Poland.

Winsorizing is a method where reduction to a deviation from the mean is applied. For every contestant, the mean of jurorsâ€™ scores M s is computed. There is established a deviation from the mean, a. The deviation is common for all contestants. If a score is lower than $M-a$, it is increased to $M-a$. If a sore is higher than $M+a$, it is reduced to $M+a$. The other scores remained unchanged. The mean of such improved scores is computed, and the contestant with the highest mean wins. This method and its generalizations were used during the International Fryderyk Chopin Piano Competitions, one of the main piano competitions in the world.

The third method is a new method introduced by Kontek and Sosnowska [2]. For every contestant, the mean of scores is computed. Then,

for every juror, the distance between the vector of the jurorâ€™s scores and the vector of means is computed. 20% of jurors with the highest such distance are removed. The means of scores of the rest jurors are computed, and the contestant with the highest mean wins. This method was used in the International Chopin Competition for Amateurs.

The Borda Count is characterized by a set of conditions that it fulfils, and any method fulfilling these conditions is the Borda Count [4]. The same property has any scoring method [5]. It is a question whether the Olympic Mean, winsorizing, and anti-manipulation methods have such characterization. The question is open, but the properties of the method are studied. In this paper, we consider the following properties: consistency, vulnerability to the No-Show paradox, vulnerability to the Condorcet winner paradox, vulnerability to the Condorcet loser paradox, monotonicity, homogeneity, subset choice condition, and vulnerability to reversal bias. The methods are compared respectively, fulfilling these properties.

[1] Gibbard A., *A Pareto-consistent libertarian claim*, Journal of Economic Theory 7 (1974), 388–410.

[2] Kontek K., Sosnowska H., *Specific tastes or cliques of jurors. How to reduce the level of manipulation in group decisions?*, Group Decision and Negotiation 19 (2020), 1057–1084.

[3] Satterthwaite M.A., *Strategy-proofness and Arrow's condition: existence and correspondence for voting procedures and social welfare functions*, Journal of Economic Theory 10 (1975), 187–221.

[4] Young H.P., *Axiomatization of Borda's rule*, Journal of Economic Theory 9 (1974), 43–52.

[5] Young H.P., *Social Choice Scoring Functions*, SIAM Journal of Applied Mathematics 28 (1975), no. 4, 824–838.

A mathematical model of ”the tragedy of the commons” with relation to counteracting pandemic

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joint work with Rajani Singh
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Abstract

We model ”the tragedy of the commons” when a scarce resource becomes a strategic input in production of a countermeasure to a sudden threat to the society's safety. The model uses the terminology of squalene market in the context of COVID-19 vaccines with adjuvants based on squalene obtained from endangered sharks and it is a dynamic game taking into account the privileged position of a relatively small number of vaccine producers.

From the economic point of view, the game describes a market consisting of pharmaceutical, cosmetic and fishing sector, in presence of a regulatory institution. From the mathematical point of view, we face coupled and hierarchical optimization problems of all the agents, who are elements of two continua and a discrete finite set. We consider Nash and Stackelberg equilibria in which COVID-19 vaccine producers do not take into account their influence on the population of sharks. Using topological and convex analysis tools, we prove the existence and uniqueness of equilibrium together with deriving the formula to find it.

This ”tragedy of the commons” endangers the vaccination programme: either because of depletion of the shark population or its drastic reduction. Various remedies for the regulating agency are suggested.

Despite terminology related to a specific problem, this is a starting point to a general theory how to counteract such risks a priori.

A global version of Tarski-Kantorovitch theorem for correspondences

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joint work with Łukasz Balbus (U of Zielona Góra), Wojciech Olszewski (Northwestern University) and Kevin Reffett (Arizona State University)

Abstract

For a strong set order increasing (resp., strongly monotone) upper order hemicontinuous correspondence $F : A \rightrightarrows A$, where A is a complete lattice (resp., a σ-complete lattice), we provide tight fixed-point bounds for sufficiently large iterations $F^k(a^0)$, starting from any point $a^0 \in A$. Our results, hence, prove a generalization of the Tarski-Kantorovich principle. We provide application of our results to a class of social learning models on networks.

Optimal portfolio selection on the bond market

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Abstract

We assume that our investment strategy is a general signed measure distributed on all real numbers representing time to maturity specifications for different rolling bonds. A nice feature of the model is the possibility to include portfolio consisting of different coupon bonds. The dynamics of the instantaneous interest rate follows a factor model determined by a system of stochastic differential equations. We solve a dynamic consumption–investment problem (joint paper with Szymon Peszat) as well as a dynamic mean-variance problem (joint work with Jakub Trybuła).

[1] Peszat S., Zawisza D., *The investor problem based on the HJM model*, Annales Polonici Mathematici 127 (2021), 241–269.

Thematic section

MSML
Mathematics and Statistics in Machine Learning

ORGANIZERS:

Enrique Dominguez (Universidad de Malaga)

Andrzej Nowakowski (Uniwersytet Łódzki)

SCHEDULE OF THE SECTION
Mathematics and Statistics in Machine Learning

- Monday – September 4th

  16:00–16:30 Krzysztof Ślot, *Hybrid neuromorphic architecture with neural representation extraction and hyperdimensional analyzer for image and video classification*

  16:30–17:00 Krzysztof Podlaski, *Automatic method for transcription of handwritten archival documents*

  coffee break

  17:30–18:00 Domingo Lopez Rodriguez, *Explainable Machine Learning using Formal Concept Analysis*

  18:00–18:30 Arkadiusz Tomczyk, *Applications and explainability of graph neural networks*

  18:30–19:00 Kamil Kołodziejski, *Introduction to vector autoregression model*

- Tuesday – September 5th

  14:30–15:00 Bartosz Zieliński, *Learning logical rules using neural networks*

  15:00–15:30 Radosław Matusik, *Fixed-time anti-synchronization for reaction-diffusion neural networks*

  15:30–16:00 Janusz Gajda, *Neural networks boosted by fractional operators*

  coffee break

  16:30–17:00 Marta Lipnicka, *Learning of Neural Network with Optimal Control Tools*

Neural networks boosted by fractional operators

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Abstract

We will discuss the appearance of fractional operators in the context of data analysis with the use of machine learning methods. We discuss the concept of fractional (non-integer order) differentiation on real data of four datasets based on stock prices of main international stock indexes: WIG 20, S&P 500, DAX and Nikkei 225. For fractionally differenced series we use artificial neural networks (ANN) to build a predicitive model. Our work is based on the paper [1].

[1] Walasek R., Gajda J., *Fractional differentiation and its use in machine learning*, International Journal of Advances in Engineering Sciences and Applied Mathematics 13 (2021), no. 2-3, p. 270–277.

Introduction to Vector Autoregression model

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Abstract

When we want to predict future values based on historical observations, we rely on time series models. If our dataset consists of multiple time series that influence each other, we can use Vector Autoregression.

This paper introduces the VAR model [1], presents the Yule-Walker method of parameters estimation and Levinson-Durbin algorithm which solves these equations [3]. To find the optimal order of the VAR model, the Akaike Information Criterion will be shown. Next, there will be discussion about the forecasting problem using the linear predictors [2]. The presentation will be enhanced with the VAR model training examples in R programming language.

[1] Brockwell P.J., Davis R.A., *Introduction to Time Series and Forecasting*, Third edition, 2016.

[2] Brockwell P.J., Davis R.A., *Time Series: Theory and Methods*, Second Edition, 1991.

[3] Musicus B.R., *Levinson and Fast Choleski Algorithms for Toeplitz and Almost Toeplitz Matrices* (1988).

Learning of neural network with optimal control tools

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Abstract

Any learned artificial neural network on a given set of observations represents a function of several variables with vector values or real values. However, in general it is unknown except very simply cases and we have trouble to tell anything about its properties behind very general results received from learning data. In applications, such as medicine, it needs to say not only that on training data we get some error, we have to know that an error is not greater than some ε for all data for which we consider the system.

It is well known that the learned neural network define a function. However, we check correctness of it only on finite number of observed data. We develop an optimal control approach allowing to find approximation of an unknown function realizing the given observable data, parametrized by a set of controls and defined as ordinary differential equations. Moreover, to measure discrepancy, of the output of the network we define a functional into which we include probability distribution function estimating distribution of the data.

We develop a dual dynamic programming ideas to formulate a new optimization problem. We apply it to derive and to prove sufficient approximate optimality conditions for approximate neural network which should work correctly for given ε with respect to built functional, on a data different than the set of observations.

[1] Kosmatka K., Nowakowski A., *Estimating supervisor set using machine learning and optimal control*, Advances in Computational Intelligence 116 (2019), 1089–1096.

[2] Lipnicka M., Nowakowski A., *Optimal control in learning neural network*, In: Computation and Optimization in Information Systems and Management Sciences. MCO 2021. In: Lecture Notes in Networks and Systems 33, 2022.

[3] Lipnicka M., Nowakowski A., *Optimal control using to approximate probability distribution of observation set*, Mathematical Methods in the Applied Sciences 1-16 (2022), doi: 10.1002/mma.8391.

[4] Nowakowski A., *The dual dynamic programming*, Proceedings of the American Mathematical Society 116 (1992), 1089–1096.

[5] Vapnik A., *The Nature of Statistical Learning Theory*, Springer, New York, 2000.

Fixed-time anti-synchronization for reaction-diffusion neural networks

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joint work with Anna Michalak
University of Lodz, Faculty of Economics and Sociology
and Andrzej Nowakowski
University of LodzFaculty of Mathematics and Computer Science

Abstract

We consider the reaction-diffusion neural network for which coefficients and neural function depend on time and spatial variable. We study fixed-time anti-synchronization (FTAS) problem. We develop a dual dynamic programming theory to derive verification theorem allowing to find and verify the best fixed-time for anti-synchronization of the system.

[1] Chen S., Lim C.-C., Shi P., Lu Z., *Synchronization control for reaction-diffusion FitzHugh-Nagumo systems with spatial sampled data*, Automatica 93 (2018), no. 1, 352–361.

[2] Chen W.-H., Luo S., Zheng W. X., *Impulsive synchronization of reaction-diffusion neural networks with mixed delays and its application to image en- cryption*, IEEE Transactions on Neural Networks and Learning Systems 27 (2016), no. 12, 2696–2710.

[3] Chen Y., Yu W. W., Tan S. L., Zhu H. H., *Synchronizing nonlinear com- plex networks via switching disconnected topology*, Automatica 70 (2016), 189–194.

[4] Ji G., Hu C., Yu J., Jiang H., *Finite-time and fixed-time synchronization of discontinuous complex networks: A Unified control framework design*, Journal of the Franklin Institute 355 (2018), no. 11, 4665–4685.

[5] Lu L., Li C., Wang Z. et al., *Anti-synchronization transmission of the laser signal using uncertain neural network*, Optik 126 (2015), no. 22, 3385–3389.

[6] Nowakowski A., *The dual dynamic programming*, Proceedings of the American Mathematical Society 116 (1992), no. 4, 1089–1096.

[7] Rakkiyappan R., Dharani S., Zhu Q. X., *Synchronization of reaction-diffusion neural networks with time-varying delays via stochastic sampled-data controller*, Nonlinear Dynamics 79 (2015), no. 1, 485–500.

[8] Wang J.-L., Wu H.-N., Guo L., *Novel adaptive strategies for synchronization of linearly coupled neural networks with reaction-diffusion terms*, IEEE Transactions on Neural Networks and Learning Systsems 25 (2014), no. 2, 429–440.

[9] Wang Z., Cao J., Cai Z., Rutkowski L., *Anti-Synchronization in Fixed Time for Discontinuous Reaction-Diffusion Neural Networks With Time-Varying Coefficients and Time Delay*, IEEE Transactions on Cybernetics 50 (2020), no. 6, 2758–2769.

[10] Wu Y., Liu L., Hu J., Feng G., *Adaptive antisynchronization of multilayer reaction-diffusion neural networks*, IEEE Transactions on Neural Networks and Learning. Systsems 29 (2018), no. 4, 807–818.

Automatic method for transcription of handwritten archival documents

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Abstract

Digitalizing archival documents is an ongoing and important operation for preserving human heritage. Most of the existing archives store documents as well as their scanned copies. Unfortunately, the images of the documents are not suitable for easy use. These documents must be transcribed to be effectively used by people or automatic systems. In this talk, we present the usage of a Convolutional Recurrent Neural Network (CRNN) for the transcription of archives from The State Lodz Archives in Poland. The documents are handwritten in the thirties of the twentieth century. We describe the preprocessing of the images and the CRNN implementation suited for this task. We discuss the effectiveness of the transcription. It proves that the CRNN network can be used for the task, as most of the words were properly transcribed.

[1] Arthur Flor de Sousa Neto, Byron Leite Dantas Bezerra, Alejandro Hector Toselli, and Estanislau Baptista Lima. 2020. HTR-Flor: A Deep Learning System for Offline Handwritten Text Recognition. In *2020 33rd SIBGRAPI Conference on Graphics, Patterns and Images (SIBGRAPI)*. IEEE.

[2] Honggang Zhang, Jun Guo, Guang Chen, and Chunguang Li. 2009. HCL2000 - A Large-scale Handwritten Chinese Character Database for Handwritten Character Recognition. In *2009 10th International Conference on Document Analysis and Recognition*. IEEE.

Applications and explainability of graph neural networks

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Abstract

Graph data structures are to be found everywhere. Examples are social networks (consisting of people and / or things) and chemical compounds (composed of atoms). A special case of graphs are images where single points (pixels) are organized in a regular grid. Usually both graph nodes and edges are described by means of feature vectors characterizing them. As a result of which, it is possible to predict properties of a graph as a whole or properties of its separate nodes and / or edges (structural prediction). Tools, which are currently used for that purpose, are modern architectures of neural networks (convolutional and recurrent networks, transformers). Such prediction has many crucial applications, e.g.: drug discovery, recommender systems (including security systems) and vision systems. During the presentation a short introduction to neural networks operating on graphs will be given. It will focus on the problem of convolution generalization, attention mechanism as well as review of basic graph convolutional operators: Graph Convolutional Network, Graph Attention Network, Graph Transformer and Mixture Model Network. Next, two practical application areas of graph neural networks will be shown. The first one will be prediction of chemical compounds' properties. Here, not only classification and regression tasks will be considered, but the problem of metric learning will be examined as well. The second one will be devoted to computer vision. It will be argued that changing the representation of image content and its description using graph structures can be beneficial for that domain. In both cases the aspect of explainable artificial intelligence will be emphasized, where the goal is not only to have a well-generalizing model, but also to understand the working principles of the obtained networks. The discussed explanation experiments will base both on attention coefficients and on classic attribution techniques.

Learning logical rules using neural networks

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Abstract

Inductive logic programming (ILP) [1] is one of the traditional, rule-based approaches to machine learning. The purpose of numerous variations of ILP is to construct logic programs from examples which explain those examples. ILP approach has many advantages, especially when compared with deep learning: it is easy to incorporate background knowleadge, the resulting models are usually human-understandable (unless they are very large), and it can train on small number of examples (unlike deep learning models which usually require very large training sets). On the other hand, it does not scale very well (compared to neural networks) and it does not work very well with noisy data [2].

Recent years saw development of various neuro-symbolic techniques which combine rule based approaches with deep learning. For example, in differentiable ILP the neural model is interpretable as a description of a logic program (or rather a probability distribution over rules). Usually, the resulting "logic programs" use fuzzy logic (see e.g., [2], [3]).

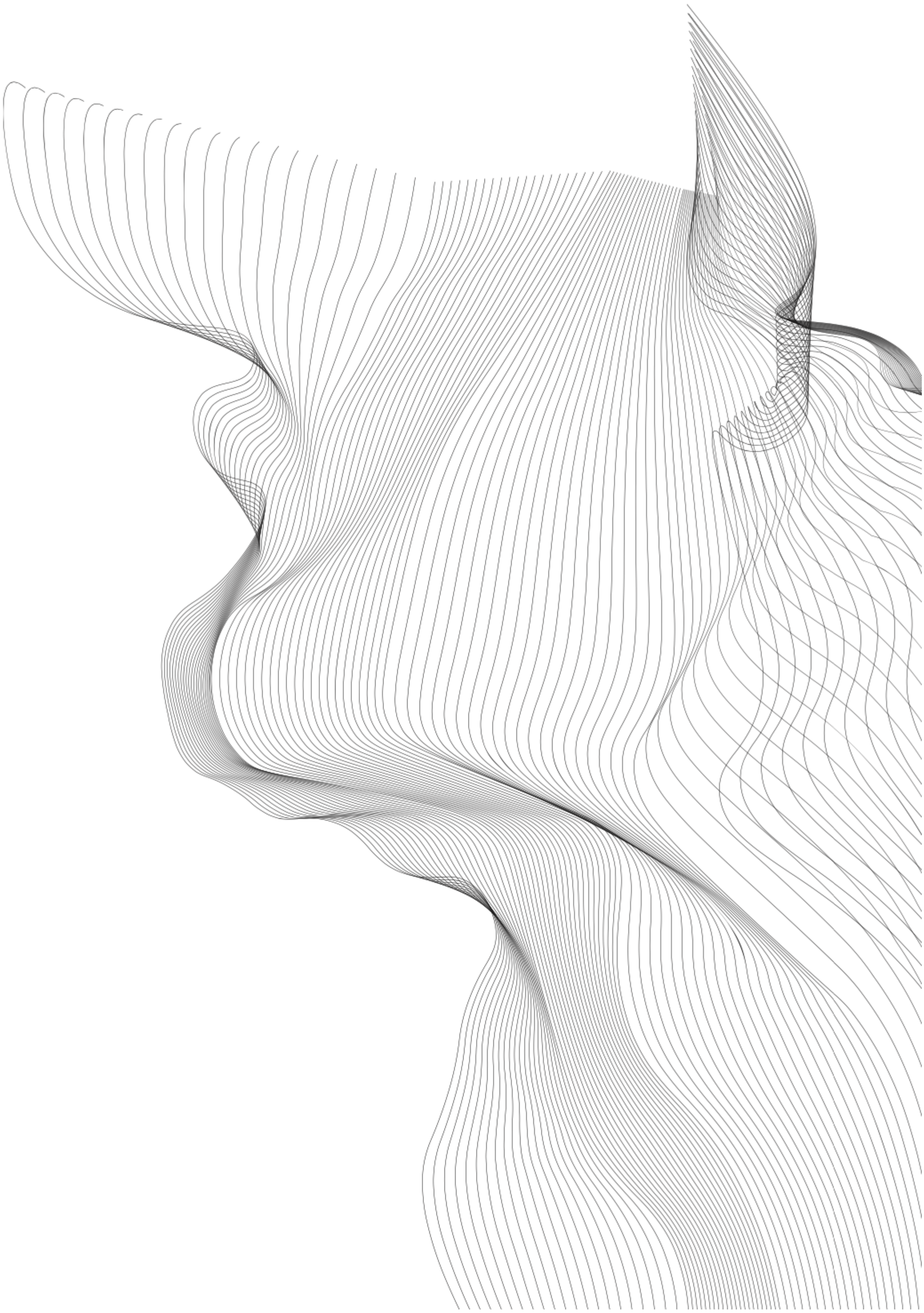
In the talk I will introduce the basics of differentiable ILP and present its applications, especially to recognition of patterns in text based on limited number of examples.

[1] Cropper A., Dumančić S., *Inductive logic programming at 30: a new introduction*, Journal of Artificial Intelligence Research 74 (2022), 765–850.

[2] Evans R., Grefenstette E., *Learning explanatory rules from noisy data*, Journal of Artificial Intelligence Research 61 (2018), 1–64.

[3] Sen P. et al., *Neuro-symbolic inductive logic programming with logical neural networks*, Proceedings of the AAAI Conference on Artificial Intelligence 36 (2022), no. 8.

[4] Susskind Z. et al., *Neuro-symbolic AI: An emerging class of AI workloads and their characterization*, arXiv:2109.06133 (2021).



Thematic section

MBM
Mathematics in Biology and Medicine

ORGANIZERS:

Juan Belmonte Beitia (Universidad de Castilla-La Mancha)

Urszula Foryś (Uniwersytet Warszawski)

SCHEDULE OF THE SECTION
Mathematics in Biology and Medicine

- Tuesday – September 5th

  14:30–15:00 Urszula Ledzewicz, *Optimal Control for a Model of the Synergy of Chemo- and Radiotherapy with Immunotherapy*

  15:00–15:30 Jesús J. Bosque, *Mathematical modelling of the hotspot of metabolic activity in cancer patients*

  15:30–16:00 Agnieszka Bartłomiejczyk, *Analysis of a mathematical model of glioma growth*

  coffee break

  16:30–17:00 Marek Bodnar, 4*D model of CAR-T therapy*

  17:00–17:30 Magdalena Szafrańska, *On the analysis of a mathematical model of car-t cell therapy for glioblastoma with logistic cancer growth*

  17:30–18:00 Beatriz Ocaña Tienda, *A mathematical perspective on brain metastases*

- Wednesday – September 6th

  12:00–12:30 Piotr Bartłomiejczyk, *One-dimensional dynamics in neuron models*

  12:30–13:00 Jacek Miękisz, *Time delays in evolutionary games*

  13:00–13:30 Radosław Wieczorek, *Multiscale stochastic individual based models*

- Thursday – September 7th

  14:30–15:00 Andrzej Nowakowski, *Optimization of HAP administration in cancer therapy*

  15:00–15:30 Zuzanna Szymańska, *Bayesian inference of a non-local proliferation model*

  15:30–16:00 Ryszard Rudnicki, *Some aspects of mathematical modelling of cell cycle*

  coffee break

  16:30–17:00 Katarzyna Pichór, *A general model of immune status*

  17:00–17:30 Marcin Choiński, *A discrete SIS model built on the strictly positive scheme*

  17:30–18:00 Jan Poleszczuk, *Non-invasive estimation of patient-specific cardiovascular system properties using mathematical modeling coupled with tonometry data*

Analysis of a mathematical model of glioma growth

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joint work with Marek Bodnar, Magdalena U. Bogdańska
and Monika J. Piotrowska

Abstract

Low-grade gliomas (LGGs) are primary brain tumours which evolve very slowly in time, however, inevitably cause patient death. In [1], we consider a PDE version of the previously proposed ODE model that describes the changes in the densities of functionally alive LGGs cells and cells that are irreversibly damaged by chemotherapy treatment. Based on the Fenichel invariant manifold theory we show that the tumour spreads like a travelling wave, meaning that the solutions of a corresponding mathematical model move with a constant speed without changing their shape. We have also calculated analytically the minimum speed of the travelling wave which is very close to the numerically calculated speed.

[1] Bartłomiejczyk A., Bodnar M., Bogdańska M.U., Piotrowska M.J., *Travelling waves for low-grade glioma growth and response to chemotherapy model*, submitted.

One-dimensional dynamics in neuron models

Piotr Bartłomiejczyk

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joint work with Frank Llovera Trujillo
and Justyna Signerska-Rynkowska

Abstract

Map-based neuron models are useful tools in modelling neural dynamics. They provide an alternative to usually computationally more expensive models based on continuous or hybrid dynamical systems. We study two discrete models of neuronal dynamics. The first model was introduced by Chialvo in 1995 ([1]) and the second one by Courbage, Nekorkin and Vdovin in 2007 ([2]). We show that their reduced one-dimensional versions can be treated as independent simple models of neural activity, which still display very rich and varied dynamics. We carry out a detailed analysis of both periodic and chaotic behaviour of the models.

[1] Bartłomiejczyk P., Llovera Trujillo F., Signerska-Rynkowska J., *Periodic and chaotic dynamics in a map-based neuron model*, Mathematical Methods in the Applied Sciences (2023), 1–26.

[2] Bartłomiejczyk P., Llovera Trujillo F., Signerska-Rynkowska J., *Spike patterns and chaos in a map-based neuron model*, to appear in International Journal of Applied Mathematics and Computer Science.

Mathematical modelling of the hotspot of metabolic activity in cancer patients

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Abstract

The collective signature emerging from the transition of cancers to increasingly aggressive behaviors is macroscopically displayed by positron emission tomography (PET). In fact, the most readily PET measure, the maximum standardized uptake value ($\mathrm{SUV}_{\mathrm{max}}$), has been found to have prognostic value in different cancers. However, few works have linked the properties of this metabolic hotspot to cancer evolutionary dynamics. By analysing diagnostic PET images from cancer patients, we found that $\mathrm{SUV}_{\mathrm{max}}$ scales superlinearly with the mean metabolic activity ($\mathrm{SUV}_{\mathrm{mean}}$) and sublinearly with the metabolic tumor volume (MTV) following power laws. This preferential accumulation of activity on the hotspot was accurately captured by a mechanistic model of tumor growth accounting for phenotypic transitions, what suggests that non-genetic changes may suffice to fuel the observed sustained increases in tumor metabolic activity. The model also revealed that the location of increasingly active proliferative cellular spots progressively drifts from the center of the tumor to the periphery, as a result of the competition between gradually more aggressive phenotypes. This last finding led to the development of a metric, NHOC (normalised distance from hotspot to centroid), based on the separation from the location of the activity hotspot to the tumor centroid. We computed the NHOC on the 3D diagnostic PET images from patients with lung cancer and patients with breast cancer by measuring the volume normalised distance from the $\mathrm{SUV}_{\mathrm{max}}$ to the tumour centroid. In both cohorts, we carried out survival analyses for the NHOC and for other classical PET biomarkers, finding that the former had a high prognostic value, outperforming the latter.

[1] Bosque J.J., Calvo G.F., Molina-García D., Pérez-Beteta J., García Vicente A.M., Pérez-García V.M., *Metabolic activity grows in human cancers pushed by phenotypic variability*, iScience 26 (2023), no. 3, 106–118.

[2] Jiménez-Sánchez J., Bosque J.J., Jiménez Londoño G.A., Molina-García D., Martínez A., Pérez-Beteta J., Ortega-Sabater C., Honguero Martínez A.F., García Vicente A.M., Calvo G.F., Pérez-García V.M., *Evolutionary dynamics at the tumor edge reveal metabolic imaging biomarkers*, PNAS 118 (2021), no. 6.

A discrete SIS model built on the strictly positive scheme

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Abstract

I will present a model which is a discretization of its continuous counterpart. As a discretization method, the strictly positive scheme was chosen. I will present the basic properties of the system, including the value of the basic reproduction number $\mathcal{R}_0$ and the existence of stationary states appearing in the system. Local stability of the stationary states will be discussed. I will also focus on global stability of the state for which there is no infection in the population. Moreover, the behavior of the system for $\mathcal{R}_0 = 1$ will be discussed. Theoretical results will be complemented with numerical simulations. They constitute a continuation of the work presented in [1] and [2].

[1] Bodzioch M., Choiński M., Foryś U., *Simple Discrete SIS Criss-Cross Model of Tuberculosis in Heterogeneous Population of Homeless and Non-Homeless People*, Mathematica Applicanda 47 (2019), no. 1, 103–115.

[2] Bodzioch M., Choiński M., Foryś U., *A Non-standard Discretized SIS Model of Epidemics*, Mathematical Biosciences and Engineering 19 (2022), no. 1, 115–133.

Optimal Control for a Model of the Synergy of Chemo- and Radiotherapy with Immunotherapy

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Abstract

The release of tumor antigens during traditional cancer treatments (such as chemo- or radiotherapy) leads to a stimulation of the immune response which provides synergistic effects these treatments have when combined with immunotherapies (e.g., based on check-point blockade). Based on a classical model for tumor-immune system interactions [2], a low-dimensional mathematical model is formulated that incorporates such synergistic features [3]. The resulting dynamics exhibits the wide range of behaviors that encompass the variety of medically realistic scenarios often termed the three E's (elimination, equilibrium and tumor escape) of immunoediting [1].

Optimal control problems for the scheduling of combinations of chemo- and immunotherapy are formulated and analyzed (both analytically and numerically) [4]. Side effects of the drugs are measured indirectly by including the total doses of the respective drugs with weights as penalty terms in the objective. The formulation allows us to judge the amounts of the agents required to achieve tumor eradication as well as the time it will take to do so. Various medical scenarios reflected through different

weights in the penalty terms are considered and solutions are computed numerically and their local optimality is verified [5].

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[3] Ledzewicz U., Schättler H., *On modeling the synergy of Cancer immunotherapy with radiotherapy*, Communications in Nonlinear Science and Numerical Simulation 118 (2023).

[4] Ledzewicz U., Schättler H., *On the optimal control problem for a model of the synergy of chemo- and immunotherapy*, Optimal Control Applications and Methods, in revision.

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Time delays in evolutionary games

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Abstract

It is well known that time delays can cause oscillations in dynamical systems. Usually, the internal equilibria of evolving populations, describing the coexistence of strategies, are expected to be asymptotically stable for small time delays, while above the critical time delay at which the Hopf bifurcation occurs, they become unstable, cycles appear. Here we present a new behavior of systems with time delays, not present in any previous models of evolutionary games [1]. We show that in differential replicator equations with strategy-dependent time delays, interior stationary states, describing the level of cooperation in evolutionary games of social dilemmas, depend continuously on time delays. We also show that they may disappear or additional states can emerge. We develop small time-delay approximation to replicator dynamics [1]. One particular example of our results is that in the Prisoner's Dilemma game, for time delays of cooperation smaller than ones of defection, an unstable interior state appears, so for some initial conditions, the population converges to a homogeneous state with just cooperators. We will also discuss some results for finite populations [2].

Acknowledgments This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 955708.

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[2] Miękisz J., Mohamadichamgavi J., *Small time delay approximation in replicator dynamics*, preprint, doi:2303.08200.

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Optimization of HAP administration in cancer therapy

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Abstract

Solid tumors are often characterized by areas with reduced oxygen levels called hypoxia. Such regions arise when tumor vascular perfusion is limited or the interstitial diffusion is perturbed, or the rapidly expanding tumors increase the overall oxygen absorption [1]. Hypoxia is one of the causes of tumor resistance to anti-cancer treatments, including chemo-, immuno-, and radiation therapies. Therefore, new treatments are being developed that can be chemically activated only in the areas of low oxygen contents – the hypoxia-activated pro-drugs (HAPs) [2]. Such a drug is usually supplied in an inactive form (a pro-drug) that requires chemical activation to generate its lethal form (an effector drug). While several HAPs have been tested in clinical trials, they were not as successful as expected [3]. Thus, more studies are needed to investigate the methods of HAP administration in order to optimize their effectiveness.

To design optimal strategies for HAP administration, we will use the following mathematical model. Let us define a rectangular domain $\Omega = \{\mathbf{x} = (x, y) : 0 \leqslant a \leqslant x \leqslant b,\ 0 \leqslant c \leqslant y \leqslant d\}$ and boundary of Ω denoted as $\partial\Omega = O_l \cup O_r$ where $O_l = \{(a, y) : y \in [c, d]\}$, $O_r = \partial\Omega \backslash O_l$. Inside the domain Ω, we consider N tumor cells Γ_l, $l = 1, ...N$, each with N_l boundary points $\mathbf{X}_i^l$, $l = 1, ...N_l$. The domain Ω is interpenetrated by the interstitial fluid flow of velocity $\mathbf{u}(\mathbf{x})$. The flow is stationary, since over the time of this project (3 hours), the cells are assumed to be non-motile and non-proliferating.

The spatial distribution of an inactive prodrug η_i is described by the following diffusion-advection-reaction equation with a constant diffusion coefficient D_{η_i}, for $(\mathbf{x}, t) \in \Omega \times [0, T]$

$$\frac{\partial \eta_i(\mathbf{x}, t)}{\partial t} = D_{\eta_i} \Delta \eta_i(\mathbf{x}, t) + \mathbf{u}(\mathbf{x}) \cdot \nabla \eta_i(\mathbf{x}, t) - \varphi(\gamma(\mathbf{x}, t))\eta_i(\mathbf{x}, t),$$

where φ is the drug activation function that is non-zero ($\varphi_0 > 0$) only in the hypoxic areas, given by

$$\varphi(\gamma(\mathbf{x},t)) = \begin{cases} \varphi_0 & \text{if } \gamma(\mathbf{x},t) \leqslant \gamma_{hypo} \\ 0 & \text{otherwise} \end{cases}$$

the initial condition is given by

$$\eta_i(\mathbf{x},0) = 0, \ \mathbf{x} \in \Omega$$

and boundary conditions describe the pro-drug bolus injection (over time $[0,t_0]$) that defines influx from the boundary ($y \in [c,d]$) representing the tumor vessel, are defined as follows

$$\eta_i(a,y,t) = \begin{cases} w_2(y,t), & \text{if } y \in [c,d], \ t \in [0,t_0], \ t_0 < T, \\ 0 & \text{if } y \in [c,d], \ t \in (t_0,T], \end{cases}$$

$$\frac{\partial \eta_i(x,y,t)}{\partial \nu} = 0, \ (x,y) \in O_r, \ t \in [0,T],$$

The spatial distribution of oxygen γ is described by the following diffusion-advection-reaction equation with a constant diffusion coefficient D_γ and an oxygen uptake rate β, for $(\mathbf{x},t) \in \Omega \times [0,T]$

$$\frac{\partial \gamma(\mathbf{x},t)}{\partial t} = D_\gamma \Delta \gamma(\mathbf{x},t) + \mathbf{u}(\mathbf{x}) \cdot \nabla \gamma(\mathbf{x},t) - \beta \gamma(\mathbf{x},t) \sum_{l=1}^{N} \sum_{i=1}^{N_l} \chi_\varepsilon(\mathbf{x}, \mathbf{X}_i^l),$$

here, the indicator function χ_ε defines the neighborhood of radius ε for information transfer between cells and the domain where the metabolite distributions are defined

$$\chi_\varepsilon(\mathbf{x}, \mathbf{X}_i^l) = \begin{cases} 1 & \text{if } \|\mathbf{x} - \mathbf{X}_i^l\| \leqslant \varepsilon \\ 0 & \text{otherwise} \end{cases}$$

the initial condition for oxygen concentration represents a non-uniform oxygen gradient and is given by

$$\gamma(\mathbf{x},0) = \gamma_0(\mathbf{x}), \ \text{for } \mathbf{x} \in \Omega$$

and the boundary conditions are given by

$$\gamma(a,y,t) = w_1(y,t), \text{for } y \in [c,d], \ t \in [0,T],$$

$$\frac{\partial \gamma(\mathbf{x},t)}{\partial \nu} = 0, \ \text{for } \mathbf{x} \in O_r, \ t \in [0,T],$$

The spatial distribution of an active drug η_a is described by the following diffusion-advection-reaction equation with a constant diffusion coefficient D_{η_a}, the drug activation function φ, the drug uptake rate α, and a drug decay rate ω_a,

$$\begin{aligned} \frac{\partial \eta_a(\mathbf{x},t)}{\partial t} &= D_{\eta_a} \Delta \eta_a(\mathbf{x},t) + \mathbf{u}(\mathbf{x}) \cdot \nabla \eta_a(\mathbf{x},t) + \varphi(\gamma(\mathbf{x},t)) \eta_i(\mathbf{x},t) \\ &\quad - \alpha \eta_a(\mathbf{x},t) \sum_{l=1}^{N} \sum_{i=1}^{N_l} \chi_\varepsilon(\mathbf{x}, \mathbf{X}_i^l) - \omega_a \eta_a(\mathbf{x},t), \\ &\qquad \text{for } (\mathbf{x},t) \in \Omega \times [0,T] \end{aligned}$$

under initial condition

$$\eta_a(\mathbf{x}, 0) = 0, \text{ for } \mathbf{x} \in \Omega$$

and boundary conditions

$$\eta_a(a, y, t) = 0, \text{ for } y \in [c, d], \ t \in [0, T]$$

$$\frac{\partial \eta_a(\mathbf{x}, t)}{\partial \nu} = 0, \text{ for } \mathbf{x} \in O_r, \ t \in [0, T],$$

[1] Brown J.M., Wilson W.R., *Exploiting tumour hypoxia in cancer treatment*, Nature Reviews Cancer 4 (2004), no. 6, 437–47.

[2] Hay M.P., Wilson W.R., *Targeting hypoxia in cancer therapy*, Nature Reviews Cancer, 11 (2011), no. 6, 393–410.

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A mathematical perspective on brain metastases

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Abstract

Brain metastases (BMs) are a major clinical problem, as they represent the most common intracranial tumors in adults. These tumors are caused by single tumor cells or groups of cells that detach from the primary site and migrate to the brain, where they give rise to clusters of metastatic cells that grow and form macrometastases.
Understanding the underlying mechanisms of BM formation is crucial for the development of effective therapeutic strategies. To shed light on this process, we have used a discrete agent-based model (ABM). Our ABM is based on a fixed vasculature and cells that are characterized by a continuous phenotype variable ranging from 0 to 1. In this model, cells can proliferate, migrate, undergo cell death, or become quiescent, resulting in the emergence of complex phenomena and the formation of large metastases.
We also wanted to study the growth dynamics of BMs. To do that we have employed scaling laws, specifically a reduced form of the Von-Bertalanffy equation

$$\frac{dV}{dt} = \alpha V^{\beta},$$

to describe the growth patterns of these tumors before and after treatment [1]. Interestingly, we have found that different growth patterns are observed for radiation necrosis, an adverse event that frequently occurs after irradiation and tumor recurrence [2]. This distinction can be useful in the clinical setting to differentiate between these two conditions, which is a clinical issue at the moment since both events have the same characteristics in medical images and however, completely different actions should be taken.
To further validate these findings, mathematical models incorporating aspects of tumor biology and inflammatory response have been developed. The results obtained from these models are consistent with the observations made in the data.

[1] Jiménez-Sánchez J., Ocaña-Tienda B., Pérez-Beteta J. et al., *The Growth Laws of Brain Metastases*, medRxiv:10.1101/2022.02.03.22270146v2

[2] Molina-García D., Ocaña-Tienda B., Pérez-Beteta J. et al., *Growth dynamics of brain metastases differentiate radiation necrosis from recurrence*, Neurooncology Advances 5 (2022), no. 1.

A general model of immune status

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Abstract

The immune status is the concentration of specific antibodies, which appear after infection with a pathogen and remain in serum, providing protection against future attacks of that same pathogen. Over time the number of antibodies decreases until the next infection. During an infection, the immunity is boosted and then the immunity is gradually waning, etc. The densities of antibody concentration satisfy some partial differential equation with an integral boundary condition, which generates a stochastic semigroup. We present general results concerning asymptotic stability and sweeping of stochastic semigroups [1] and then we apply them to our model [2]. We also analyze special cases of the model, e.g. when immunity decreases exponentially; with constant increase of antibodies after infection; with a threshold concentration of antibodies at the re-infection; and with seasonal infections.

[1] Pichór K., Rudnicki R., *Asymptotic decomposition of substochastic operators and semigroups*, Journal of Mathematical Analysis and Applications 436 (2016), 305–321.

[2] Pichór K., Rudnicki R., *Asymptotic properties of a general model of immune status*, SIAM Journal on Applied Mathematics (SIAP) 83 (2023), 172–193.

Non-invasive estimation of patient-specific cardiovascular system properties using mathematical modeling coupled with tonometry data

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Abstract

Assessment of detailed status of cardiovascular (CV) system state in hemodialysis patients is of utmost importance as the risk of CV associated death is in that group the highest among all other comorbidities. The aim of our study was to develop a mathematical model which, after calibration with patient-specific data, would provide new personalized information about CV system state.

We model the blood transport in a bifurcating binary tree of fifty-five larger systemic arteries in which individual vessels are axisymmetric elastic cylinders tapering along their length. We describe spatiotemporal changes in the cross-sectional area of the artery (equivalently blood pressure) and blood flow using an 1D approach. Proposed model was confronted with radial pressure wave profiles recorded before, during and after two independent hemodialysis sessions in 35 anuric prevalent hemodialysis patients and once in a group of 32 healthy volunteers. Each recording was used to estimate six subject-specific parameters of pulse wave propagation model.

The model identified increased arterial stiffness of both large and small arteries in hemodialysis patients compared to their healthy counterparts. Interestingly, regular pulse wave analysis based biomarkers failed to show significant differences.

Our study shows that, after parameter estimation procedure, proposed mathematical model is able to provide new patient-specific insights into CV system state that are unattainable with existing non-invasive methods.

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A mathematical model of the interplay of labile and glycated hemoglobin with glucose for clinical and research applications

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Abstract

Diabetes mellitus continues to be a major global burden, and innovations remain necessary in its early diagnosis and treatment [1]. Glycated hemoglobin (HbA1c) is an established indicator of the average glucose levels of a given patient during the two months prior to its measurement. In our project, we combine dimensionality reduction alongside an ordinary differential equation model to analyze the relationship of labile hemoglobin (HbA1d), an unstable molecule and an intermediary step in cell glycation, with the glucose and HbA1c levels in patients. It has historically only been studied as a source of error in HbA1c measurements but has recently gained interest as a biomarker of diabetes-related malignancies and an indicator of the glycemic state of patients [1], [2]. These methods are applied to two different sets of patients: a group of sixty pediatric patients on which a glucose tolerance test was performed alongside concurrent measurements of their HbA1d and HbA1c, and a group of forty thousand adult patients in which the fast- ing plasma glucose (FPG), HbA1c, and HbA1d were measured once. Through this study, we managed to find a close relationship between the dynamics of HbA1d and glucose in short time intervals and develop a predictive mathematical model that may serve to assist in the personalized treatment of individual diabetic patients. Furthermore, we examined labile hemoglobin as an indicator of diabetes in patients in whom both FPG and HbA1c give inconclusive results.

These results and models are both analyzed mathematically and tested for their biological validity. This is done by exploring the stability of their solutions and the biochemical mechanisms that dictate cell glycation, alongside their ability to predict and fit the clinical data used in this project [4].

[1] Delanghe J.R. et al., *Labile glycated hemoglobin: an underestimated laboratory marker of short term glycemia*, Clinical Chemistry and Laboratory Medicine 60 (2022), no. 3, 451–455.

[2] León-Triana O. et al., *Labile haemoglobin as a glycaemic biomarker for patient-specific monitoring of diabetes: mathematical modelling approach*, Journal of the Royal Society Interface 15 (2018), no. 142.

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[4] Sun H. et al., *Global, regional and country-level diabetes prevalence estimates for 2021 and projections for 2045*, Diabetes Research and Clinical Practice 183 (2022), 109–119.

Some aspects of mathematical modelling of cell cycle

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Abstract

Modeling of cell cycle is one of the fundamental subject of mathematical biology because it could help to solve such problems as synchronization of cell division in cancer therapy and allows to understand dynamics of growth of cellular populations (e.g. tissues). There are many different models of cell cycles. In this talk we consider an age-size structured cell population model based on the cell cycle length [1]. The model is described by a first order partial differential equation with initial-boundary conditions. Using the theory of semigroups of positive operators we establish new criteria for an asynchronous exponential growth of solutions to such equations. We discuss the question of exponential size growth of cells. We study in detail a constant size growth model and a model with target size division. We also present versions of the model when the population is heterogeneous. The discussion on model generalizations will be a good excuse to present some new challenges in the study of asymptotic behaviour of semigroups of operators.

[1] Pichór K., Rudnicki R., *Cell cycle length and long-time behaviour of an age-size model*, Mathematical Methods in the Applied Sciences 45 (2022), 5797–5820.

On the analysis of a mathematical model of CAR-T cell therapy for glioblastoma with logistic cancer growth

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Abstract

In the era of new technologies and rapidly developing medicine, scientists are trying to find better and better methods of cancer therapy. In my presentation, I will discuss the analysis of a mathematical model of CAR-T cell therapy for glioblastoma with logistic tumor growth.

The main purpose of the analysis is to determine model steady states and to study their stability. In addition, numerical simulations were performed for two cases: a single injection of CAR-T cells, which is reflected in the initial state (dose) and the case of a constant influx of CAR-T cells, which is the simplest mathematical approximation of treatment. Mathematical analysis with simulations allows us to better understand the dynamics of glioblastoma development under the influence of CAR-T cell therapy, what can help in the development of more effective therapeutic strategies.

[1] León-Triana O., Perez-Martinez A., Perez-Garcia V., *Dual-target car-ts with on- and off-tumour activity may override immune suppression in solid cancers: A mathematical proof of concept*, Cancers 13 (2021), no. 4.

[2] Li J.J., Lu P.H., Zhang X., *Advances in the development of chimeric antigen receptor-T-cell therapy in B-cell acute lymphoblastic leukemia*, Chinese Medical Journal 133 (2020), no. 4, 474-482.

Bayesian inference of a non-local proliferation model

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Abstract

We present a new proliferation model of cells living within a colony that is a non-local equation with a discontinuous interaction kernel. We discuss the range of applicability of the model, select suitable data and apply the Bayesian method to perform parameter estimation. We discuss proof of the well-posedness of the problem and we investigate the convergence of the EBT algorithm applied to solve the equation. The main difficulty lies in the low regularity of the kernel which is not Lipschitz continuous, thus preventing the application of standard arguments. Therefore, we use the radial symmetry of the problem instead and transform it using spherical coordinates. The resulting equation has a Lipschitz kernel with only one singularity at zero. We introduce a new weighted flat norm and prove that the particle method converges in this norm. We prove the well-posedness of the problem and we investigate the convergence of the EBT algorithm applied to solve the equation. Finally, we prove the stability of posterior distributions in the total variation norm which exploits the theory of spaces of measures equipped with the weighted flat norm.

[1] Gwiazda P., Miasojedow B., Skrzeczkowski J., Szymańska Z., *Convergence of the EBT method for a non-local model of cell proliferation with discontinuous interaction kernel*, IMA Journal of Numerical Analysis 43 (2023), no. 1, 590–626.

[2] Gwiazda P., Miasojedow B., Skrzeczkowski J., Szymańska Z., *Bayesian inference of a non-local proliferation model*, Royal Society Open Science 8 (2021), no. 11, 211–279.

Multiscale stochastic individual based models

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Abstract

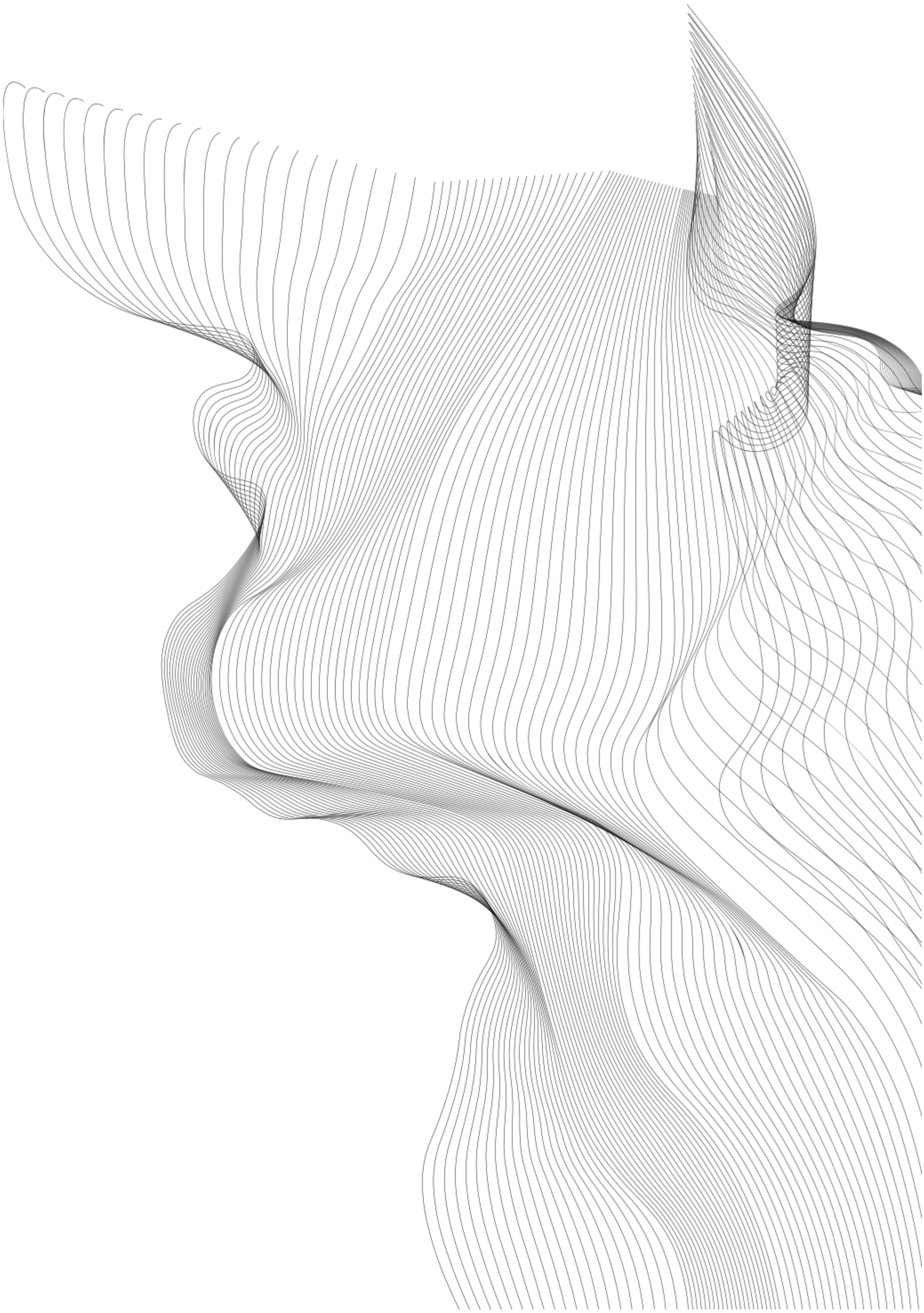
Individual-based models or stochastic particle models are very common in modern mathematical modelling, especially in biology and chemistry. Behavior of such systems when the number of particles is big is interesting both from mathematical and application point of view.

In the talk we will consider a situation when one of the population is abundant enough to use macroscopic approximation, while the other consist of a few particles and is described by a stochastic particle system. Such a scale separation leads to the so called hybrid models, where a stochastic particle system is coupled to partial differential equation. Some examples of such models and related limit theorems will be presented.

[1] Capasso V., Wieczorek R., *A hybrid stochastic model of retinal angiogenesis*, Mathematical Methods in the Applied Sciences 43 (2020), no. 18, 10578–10592.

[2] Wieczorek R., *Hydrodynamic limit of a stochastic model of proliferating cells with chemotaxis*, Kinetic and Related Models, 16 (2023), 373–393.

[3] Wieczorek R., *Multiscale reaction-diffusion stochastic particle models*, in preparation.



Thematic section

OA

Operator Algebras

ORGANIZERS:

Joan Bosa (Universidad de Zaragoza)

Stanisław Goldstein (Uniwersytet Łódzki)

Bartosz Kwaśniewski (Uniwersytet w Białymstoku)

Francesc Perera (Universitat Autonoma de Barcelona)

Mateusz Wasilewski (Instytut Matematyczny PAN)

SCHEDULE OF THE SECTION
Operator Algebra

- Monday – September 4th

  16:00–16:30 Adam Paszkiewicz, *Linear combinations of projections and perturbations of projections in von Neumann factors of type* II_1

  16:30–17:00 Rafał Wieczorek, *Equality of entropies for states*

  coffee break

  17:30–18:15 Piotr Koszmider, *On singular pure states*

  18:15–18:45 Żywilla Fechner, *Moment functions and derivations on hypergroups*

- Tuesday – September 5th

  14:30–15:15 Antonio M. Peralta, *Metric invariants for unital Banach and Jordan–Banach algebras*

  15:15–16:00 Jan Hamhalter, *Maps preserving products of commuting elements between important structures on operator algebras*

  coffee break

  16:30–17:15 Bartosz Kwaśniewski, *Topologically free actions and ideals in twisted Banach algebra crossed products*

  17:15–17:45 Aleksandra Świątczak, *Delta-Sincov mappings in Banach algebras*

- Thursday – September 7th

  14:00–14:45 Joan Claramunt Carós, *A graph-theoretic characterization of a class of dynamical systems and its (C^*)-algebras*

  14:45–15:30 Adam Skalski, *Characterising residually finite dimensional C^*-algebras in dynamical contexts*

  15:30–16:00 Tomasz Brzeziński, *The C^*-algebras of quantum lens and weighted projective spaces*

  coffee break

  16:30–17:15 Mateusz Wasilewski, *Quantum Cayley graphs*

The C^*-algebras of quantum lens and weighted projective spaces

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based on a joint work with Wojciech Szymański (Odense)

Abstract

We identify the algebras of continuous functions on quantum weighted projective and lens spaces as graph C^*-algebras. This is then used to compute the K-theory of these spaces in cases that have not been computed previously.

[1] Brzeziński T., Szymański W., *The C^*-algebras of quantum lens and weighted projective spaces*, Journal of Noncommutative Geometry 12 (2018), 195–215.

A graph-theoretic characterization of a class of dynamical systems and its (C^*)-algebras

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joint work with Pere Ara (Universitat Autònoma de Barcelona)

Abstract

We present a graph-theoretic model for dynamical systems given by a surjective local homeomorphism on a totally disconnected compact metric space. This construction gives a bijective correspondence between such dynamical systems and a subclass of two-colored Bratteli separated graphs.

We use this construction in order to write any dynamical system of our interest as an inverse limit of a sequence of (what we call) *generalized finite shifts*. This enables us to compute the associated Steinberg algebra (resp. C^*-algebra) of the dynamical system as colimits of the graph algebras (resp. graph C^*-algebras) associated with the different levels of the corresponding separated graph.

In subsequent work we plan to apply this theory to relate the type semigroup of the dynamical system with the graph monoid of the corresponding separated graph, and with the non-stable K-theory of the Steinberg algebra.

Moment functions and derivations on hypergroups

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joint work with Eszter Gselmann and László Székelyhidi

Abstract

The aim of this talk is to define higher order derivations and generalized moment generating functions in the hypergroup settings. The connection between these notions is given in the following theorem:

Let X be a commutative hypergroup and r a positive integer. The family $(D_\alpha)_{\alpha\in\mathbb{N}^r}$ of self-mappings on $\mathcal{M}_c(X)$ is a continuous higher order derivation of order r if and only if there exists a generalized moment function sequence $(\varphi_\alpha)_{\alpha\in\mathbb{N}^r}$ of rank r such that

$$\langle D_\alpha \mu, f\rangle = \int_X f \cdot \varphi_\alpha \, d\mu$$

holds for each μ in $\mathcal{M}_c(X)$, f in $\mathcal{C}(X)$ and α in $\mathbb{N}^r$.

The proof can be found in [2].

[1] Bloom W.R., Heyer H., *Harmonic analysis of probability measures on hypergroups*, de Gruyter Studies in Mathematics, vol. 20, Walter de Gruyter & Co., Berlin 1995.

[2] Fechner Ż, Gselmann E., Székelyhidi L., *Moment functions and exponential monomials on commutative hypergroups*, Aequationes Mathematicae 95 (2021), 1281–1290.

[3] Fechner Ż, Gselmann E., Székelyhidi L., *Moment Functions on Groups*, Results in Mathematics 76 (2021), no. 4.

Maps preserving products of commuting elements between important structures on operator algebras

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Abstract

Bijective bicontinuous maps preserving products of commuting elements (in both directions) that act between positive definite cones of von Neumann algebras or between unitary groups of von Neumann algebras are described in terms of the following parameters: (i) Jordan *-isomorphism between given algebras (ii) fixed central element (iii) appropriate form of central valued character. Especially, in the case of unitary groups of von Neumann factors, such maps are precisely either Jordan *-isomorphisms or Jordan *-isomorphisms composed with the inverse map $u \to u^{-1}$. These advances lead to new complete Jordan invariants of operator algebras. Besides, we present related general results on commutative parts of operator algebras and Banach algebras.

[1] Hamhalter J., *Piecewise *-homomorphisms and Jordan maps on C*-algebras and factor von Neumann algebras*, Journal of Mathematical Analysis and Applications 462 (2018), 1014–1031.

[2] Hamhalter J., *Maps preserving products of commuting elements in von Neumann algebras*, Journal of Mathematical Analysis and Applications 523 (2023).

[3] Molnár L., *General Mazur-Ulam type theorems and some applications*, in "Operator Semigroups Meet Complex Analysis, Harmonic Analysis and Mathematical Physics", Arendt W., Chill R., Tomilov Y. (Eds.), Operator Theory: Advances and Applications 250, 311–342, Birkhäuser, 2015.

On singular pure states

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Abstract

Let $\mathcal{B}(\ell_2)$ stand for the C*-algebra of all bounded linear operators on ℓ_2 and $\mathcal{D}_E(\ell_2)$ for the atomic masa of the operators which are diagonal with respect to an orthonormal basis E.

The relation between pure states of $\mathcal{B}(\ell_2)$ and the pure states of the algebras $\mathcal{D}_E(\ell_2)$ attracted interest at least since the seminal paper [3] of Kadison and Singer . In [2] J. Anderson conjectured that every pure state ϕ on $\mathcal{B}(\ell_2)$ is diagonalizable, that is, can be simply described by a choice of an orthonormal basis $E = (e_n)_{n\in\mathbb{N}}$ for ℓ_2 and a pure state u of $\mathcal{D}_E(\ell_2)$, i.e., for every $T \in \mathcal{B}(\ell_2)$ it satisfies

$$\phi(T) = \lim_{n\in u} \langle T(e_n), e_n \rangle,$$

where $\lim_{n\in u} c_n = c$ means that for every $\varepsilon > 0$ there is $A \in u$ such that for every $n \in A$ we have $|c_n - c| < \varepsilon$. Here we identify the pure states of $\mathcal{D}_E(\ell_2) \equiv \ell_\infty \equiv C(\beta\mathbb{N})$ with ultrafilters on $\mathbb{N}$. The nontrival case is of singular pure states, i.e., vanishing on the ideal of all compact operators on $\mathcal{B}(\ell_2)$ (not vector states).

In [1] Akemann and Weaver assuming the continuum hypothesis (CH) constructed a pure state on $\mathcal{B}(\ell_2)$ whose restriction to any masa is not a pure state on this masa (equivalently, the restriction is not multiplicative). This proved that CH implies that Anderson's conjecture is false, leaving open the question if Anderson's conjecture is consistent with the usual axioms ZFC as well. We present a new construction of a non-diagonalizable pure state which does not need any additional set-theoretic assumption and so disprove Anderson's conjecture in ZFC alone ([4]). By the solution of the Kadison-Singer problem this shows that the existence of pure states which are not multiplicative when restricted to any atomic masa also does not need any additional hypothesis.

[1] Akemann C., Weaver N., *$\mathcal{B}(\mathcal{H})$ has a pure state that is not multiplicative on any masa*, Proceedings of the National Academy of Sciences of the United States of America 105 (2008), 5313–5314.

[2] Anderson J., *A conjecture concerning the pure states of $B(H)$ and a related theorem* in Operator Theory: Advances and Applications, Birkhauser, Basel, Switzerland, 1981, vol. 2, 27–43.

[3] Kadison R.V., Singer I.M., *Extensions of pure states*, American Journal of Mathematics 81 (1959), 383–400.

[4] Koszmider P., *A non-diagonalizable pure state*, Proceedings of the National Academy of Sciences of the United States of America 117 (2020), no. 52, 33084–33089.

Topologically free actions and ideals in twisted Banach algebra crossed products

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joint work with Krzysztof Bardadyn [2]

Abstract

We generalize the well known C^*-algebraic result of Kawamura-Tomiyama [3] and Archbold-Spielberg [1] for crossed products of discrete transformation groups to the realm of Banach algebras and twisted actions. Namely, we show that topological freeneess is equivalent to the intersection property for all reduced twisted Banach algebra crossed products coming from subgroups, and in the untwisted case to a generalized intersection property for a full L^p-operator algebra crossed product for some (and hence any) $p \in [1, \infty]$. This gives efficient simplicity criteria for various Banach algebra crossed products. We also use it to identify the prime ideal space of some crossed products as the quasi-orbit space of the action.

It solves some of the open problems from [4].

[1] Archbold R.J., Spielberg J.S., *Topologically free actions and ideals in discrete C^*-dynamical systems*, Proceedings of the Edinburgh Mathematical Society 37 (1993), no. 2, 119–124.

[2] Bardadyn K., Kwaśniewski B.K., *Topologically free actions and ideals in twisted Banach algebra crossed products*, arXiv:2307.01685 (2023).

[3] Kawamura S., Tomiyama J., *Properties of topological dynamical systems and corresponding C^*-algebras*, Tokyo Journal of Mathematics 13 (1990), 251–257.

[4] Phillips N.C., *Crossed products of L^p operator algebras and the K-theory of Cuntz algebras on L^p spaces*, arXiv:1309.6406 (2013).

Linear combinations of projections and perturbations of projections in von Neumann factors of type II_1

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joint work with Stanisław Goldstein

Abstract

We present recent results of the following type: For any hermitian operator $a \in \mathcal{M}$, we have $a \in \operatorname{lin}(p_1, \ldots, p_n)$ for some projections $p_1, \ldots, p_n \in \mathcal{M}$; for some hermitian operator $a \in \mathcal{M}$, $a \notin \operatorname{lin}(p_1, \ldots, p_{n-1})$ for any projections $p_1, \ldots, p_{n-1} \in \mathcal{M}$. If proves that $n = 4$ for $\mathcal{M}$ being a factor of type I_n, $n > 76$; I_∞; II_1 or II_∞ but $n = 3$ for $\mathcal{M}$ of type III.

Some relations to the theory of conditional expectations in von Neumann algebras will be explained.

Some new look at old methods in perturbations theory of operator in Hilbert space will also be presented.

Characterising residually finite dimensional C^*-algebras in dynamical contexts

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Abstract

A C^*-algebra is said to be residually finite-dimensional (RFD) when it has 'sufficiently many' finite-dimensional representations. The RFD property is an important, and still somewhat mysterious notion, admitting several equivalent descriptions and having subtle connections to residual finiteness properties of groups. In this talk I will present certain characterisations of the RFD property for C^*-algebras of amenable étale groupoids and for C^*-algebraic crossed products by amenable actions of discrete groups, extending (and inspired by) earlier results of Bekka, Exel and Loring. I will also explain the role of the amenability assumption and describe several consequences of our main theorems. Finally I will discuss some examples, notably these related to semidirect products of groups.

[1] Shulman T., Skalski A., *RFD property for groupoid C^*-algebras of amenable groupoids and for crossed products by amenable actions*, arXiv:2305.12122 (2023).

Delta-Sincov mappings in Banach algebras

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Abstract

We study solutions and approximate solutions of the multiplicative Sincov equation
$$T(f,h) = T(f,g)T(g,h)$$
for mapping T taking values in a commutative Banach algebra.

Quantum Cayley graphs

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Abstract

I will propose a definition of a quantum Cayley graph of a discrete quantum group. The first step will be to extend the framework of quantum graphs to a very restricted infinite dimensional setting, namely the infinite direct sums of matrix algebras. For discrete quantum groups the natural candidates for quantum adjacency matrices will be convolution operators against a projection. If this projection satisfies a symmetry condition and is generating, then it can be used to define a quantum Cayley graph. These quantum graphs to a large extent do not depend on the generating projection, just like for classical groups. In the last part of the talk I will present some examples.

[1] Wasilewski M., *On quantum Cayley graphs*, arXiv:2306.15315 (2023).

Equality of entropies for states

Rafał Wieczorek

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Abstract

In this talk we will deal with the Segal and Renyi entropy of a state on a semifinite or finite von Neumann algebra. We investigate the situation when two normalised states h_ρ and h_φ have equal Rényi's entropies $S_\alpha(\rho) = S_\alpha(\varphi)$ for α in some neighbourhood of 1, and when they have equal Segal's entropies $H(h_\rho + s\mathbb{1}) = H(h_\varphi + s\mathbb{1})$ or Rényi's entropies $S_\alpha(h_\rho + s\mathbb{1}) = S_\alpha(h_\varphi + s\mathbb{1})$ for some fixed α and a family of perturbed states with s in some interval $(c, d) \subset (0, +\infty)$. We will characterize these conditions.

Thematic section

ORVR

Ordered Random Variables and Reliability Theory

ORGANIZERS:

Jorge Navarro (Universidad de Murcia)

Tomasz Rychlik (Instytut Matematyczny PAN)

SCHEDULE OF THE SECTION
Ordered Random Variables and Reliability Theory

- Monday – September 4th

  coffee break

  17:30–18:00 Mariusz Bieniek, *Comparison of L-statistics as quantile estimators*

  18:00–18:30 Andrzej Okolewski, *Sharp bounds on joint distribution functions of selected order statistics for k-independent observations*

  18:30–19:00 Tomasz Rychlik, *Bounds on the Expectations of Order Statistics for the Monotone Reversed Failure Rate Distributions*

- Tuesday – September 5th

  14:30–15:00 Jorge Navarro, *Preservation of aging classes in coherent systems*

  15:00–15:30 Magdalena Szymkowiak, *Necessary and sufficient conditions for preserving transform order properties of the component lifetimes by the system lifetime*

  15:30–16:00 Félix Belzunce, *Comparison of mean residual lives for dependent random variables*

  coffee break

  16:30–17:00 Maria Longobardi, *Relations between inactivity times of systems and lifetimes of their dual systems*

  17:00–17:30 Agnieszka Goroncy, *Number of components in the three-state* **k***-out-of-n system*

  17:30–18:00 Krzysztof Jasiński, *On the number of failed components in a series-parallel system upon system failure*

Comparison of mean residual lives for dependent random variables

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Abstract

In this talk we consider two definitions of the mean residual life ordering of two random variables taking into account their mutual dependence. We prove several closure properties for the new definitions and some inferential issues are also considered for one of them.

Comparison of L-statistics as quantile estimators

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Abstract

We consider the problem of quantile estimation by suitably chosen L-statistics. For a given sample size n we determine optimal L-statistics as estimators of the quantile of a given order $p \in (0, 1)$. We use a new criterion of optimality, introduced in our paper [1], based on sharp bounds on the bias of the estimation. First we study the most popular cases of single order statistics and linear combinations of a pair of successive order statistics. Next, we study the mean square error of derived estimators and we compare them with well known L-statistics such as Kaigh-Lachenbrugh or Harell-Davis estimators.

[1] Bieniek M., Pańczyk L., *On the choice of the optimal single order statistic in quantile estimation*, Annals of the Institute of Statistical Mathematics 75 (2023), 303–333.

Number of components in the three-state k-out-of-n system

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joint work with Krzysztof Jasiński

Abstract

We consider a three-state **k**-out-of-n system with the independent and identically distributed component lifetimes. Such system and its components can function perfectly or totally fail, but can also enter a partial performance state, when they may not fail completely but their efficiency can be much reduced. Based on definitions introduced by Huang et al. (2000) and Tian et al. (2009) we focus on the random variables representing the numbers of components in such a system in respective states and compute the probability of the number of failed and partially operating components in the system in various settings.

[1] Huang J., Wu Y., Zuo M.J., *Generalized multi-state k-out-of-n: G systems*, IEEE Transactions on Reliability 49 (2000), 105–111.

[2] Tian Z., Yam R.C.M., Zuo M.J., *Multi-state k-out-of-n systems and their performance evaluation*, IIE Transactions 41 (2009), 32–44.

On the number of failed components in a series-parallel system upon system failure

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Abstract

We study reliability properties of a series-parallel system. We assume that the component lifetimes may be dependent and non-identically distributed (DNID) discrete random variables. We consider the number of failed components upon system failure. We derive the probability mass function and the expected value of this quantity, see [3] for more details. In addition, we find the conditional probabilities corresponding to this variate given some partial information about the system failure. The results correspond to those obtained by [1] and by [2].

[1] Davies K., Dembińska A., *On the number of failed components in a k-out-of-n system upon system failure when the lifetimes are discretely distributed*, Reliability Engineering and System Safety 188 (2019), 47–61.

[2] Dembińska A., Eryilmaz S., *Discrete time series-parallel system and its optimal configuration*, Reliability Engineering and System Safety 215 (2021).

[3] Jasiński K., *On the number of failed components in a series-parallel system upon system failure when the lifetimes are DNID discrete random variables*, Metrika, under review.

Relations between inactivity times of systems and lifetimes of their dual systems

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Abstract

The relation of the inactivity time of a system with structure function ϕ and the lifetime of a system with dual structure function ϕ^* is analyzed. The interesting case in which the lifetimes $T_1, \ldots, T_n$ of the components of the system S are related to the lifetimes $T_1^*, \ldots, T_n^*$ of the component of the dual system S^* by the relation $T_i^* = 1/T_i$, $i = 1, \ldots, n$, is studied in details. Some illustrative examples are given in terms of time-homogeneous load sharing models.

[1] Barlow R.E., Proschan F., *Statistical Theory of Reliability and Life Testing: Probability models*, Hold, Reinhart and Wiston, Inc. Silver Spring, MD, 1981.

[2] Buono F., Longobardi M., De Santis E., Spizzichino F., *Multivariate reversed hazard rates and inactivity times of systems*, Methodology and Computing in Applied Probability, 2021.

[3] Buono F., Longobardi M., Spizzichino F., *Inactivity times of systems and lifetimes of their dual systems*, in preparation (2023).

Preservation of aging classes in coherent systems

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Abstract

Different aging classes are used in probability theory to measure the effect of the time in the lifetime of a unit represented as a random variable. Some well known classes that represent the natural (positive) aging property are IFR (increasing failure rate), DMRL (decreasing mean residual life) and ILR (log-concave density). In all these classes the old units are worse than the young ones. When these units are placed in systems (series systems, parallel systems, k-out-of-n system) one would expect that the aging properties are transferred to the system. However, this is not always the case and we need some conditions to get these preservation properties. In this talk we will review some results in this direction and we will present some new ones.

[1] Alimohammadi M., Navarro J., *Resolving an old problem on the preservation of the IFR property under the formation of k-out-of-n systems with discrete distributions*, under review.

[2] Navarro J.: *Preservation of DMRL and IMRL aging classes under the formation of order statistics and coherent systems*, Statistics and Probability Letters 137 (2018), 264–268.

[3] Navarro J., *Introduction to System Reliability Theory*, Springer, 2022.

[4] Rychlik T., Szymkowiak M., *Signature conditions for distributional properties of system lifetimes if component lifetimes are i.i.d. exponential*, IEEE Transactions on Reliability 1–13 (2021).

Sharp bounds on joint distribution functions of selected order statistics for k-independent observations

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Abstract

We present pointwise sharp two-sided bounds on linear combinations of multivariate marginal distribution functions of order statistics based on k-independent identically distributed random variables. The model includes the model of arbitrarily dependent observations as particular case. Our results provide accurate expectation bounds for functions of selected order statistics from k-independent random variables with a finite set of values. Moreover, they lead to best-possible upper and lower bounds for the joint reliability function of any pair of semicoherent systems based on common k-independent components.

[1] Kemperman J.H.B., *Bounding moments of an order statistic when each $k-$tuple is independent*, in: Beneš V., Štěpán J., eds., Distributions with given marginals and moment problems, Dordrecht: Kluwer Academic Publishers, 1997, 291–304.

[2] Marichal J.-L., Mathonet P., Navarro J., Paroissin C., *Joint signature of two or more systems with applications to multistate systems made up of two-state components*, European Journal of Operational Research 263 (2017), 559–570.

[3] Rychlik T., *Bounds for expectations of L-estimates for dependent samples*, Statistics 24 (1992), 1–7.

Bounds on the Expectations of Order Statistics for the Monotone Reversed Failure Rate Distributions

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joint work with Agnieszka Goroncy
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Abstract

Danielak [1] and Goroncy and Rychlik [2] presented the sharp positive upper mean-variance bounds on the expectations of order statistics based on independent identically distributed random variables with the decreasing and increasing failure rates, respectively. In this paper we determine analogous evaluations in the dual cases when the parent distributions have monotone reversed failure rates.

[1] Danielak K., *Sharp upper mean-variance bounds for trimmed means from restricted families*, Statistics 27 (2003), 305–324.

[2] Goroncy A., Rychlik T., *Evaluations of expectations of order statistics and spacings based on IFR distributions*, Metrika 79 (2016), 635–657.

[3] Goroncy A., Rychlik T., *Bounds on the expectations of order statistics for the monotone reversed failure rate distributions*, submitted (2023).

Necessary and sufficient conditions for preserving transform order properties of the component lifetimes by the system lifetime

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joint work with Tomasz Rychlik
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Abstract

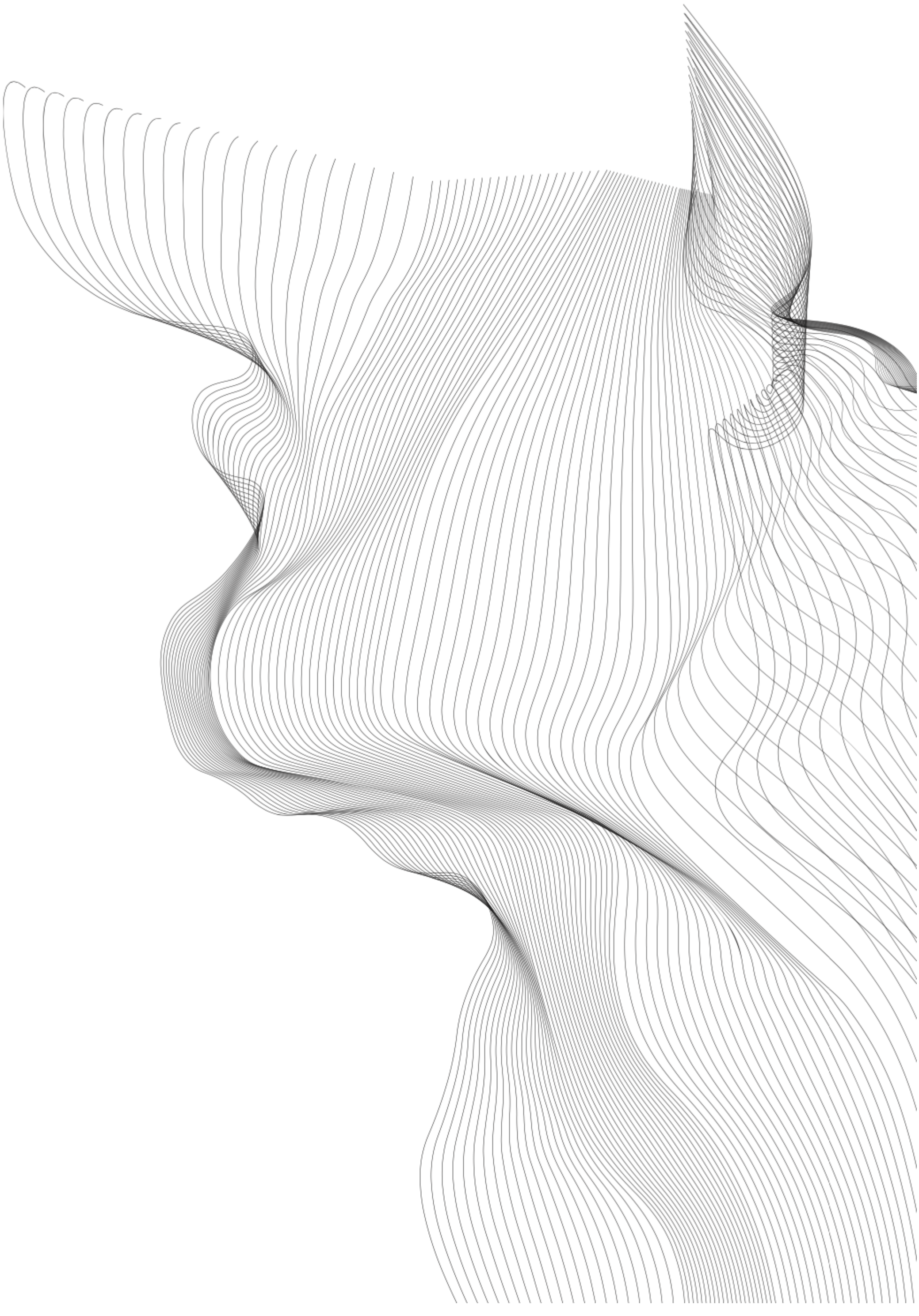
We consider coherent and mixed systems with identical components having a given structure function and a copula of dependence of component lifetimes which admits the Samaniego representation of the system lifetime distribution. We present natural necessary and sufficient conditions for preserving distributional properties of the component lifetimes by the system lifetime which are expressed by the relation of the respective distribution with some fixed life distribution in the star and superadditive orders. In particular, we provide conditions under which the system lifetime distribution preserves the IFRA, DFRA, NBU and NWU properties of component lifetimes.

[1] Arnold B.C., Rychlik T., Szymkowiak M., *Preservation of distributional properties of component lifetimes by system lifetimes*, TEST 31 (2022), 901–930.

[2] Navarro J., Rychlik T., Spizzichino F., *Conditions on marginals and copula of component lifetimes for signature representation of system lifetime*, Fuzzy Sets and Systems 415 (2021), 99–117.

[3] Rychlik T., Szymkowiak M., *Preservation of transform order properties of component lifetimes by system lifetimes*, in preparation.

[4] Shaked M., Shanthikumar J.G., *Stochastic Orders*, Springer, New York, 2007.



Thematic section

RAGS

Real Algebraic Geometry and Singularities

ORGANIZERS:

Jose Fernando Galvan (Universidad Complutense de Madrid)

Evelia Garcia Barroso (Universidad de La Laguna)

Tadeusz Krasiński (Uniwersytet Łódzki)

Wojciech Kucharz (Uniwersytet Jagielloński, Kraków)

SCHEDULE OF THE SECTION
Real Algebraic Geometry and Singularities

- Monday – September 4th

  16:00–16:30 Elías Baro, *Spectral spaces in o-minimal structures*

  16:30–17:00 Carles Bivià-Ausina, *Global multiplicity, special closure and non-degeneracy of gradient maps*

  coffee break

  17:30–18:00 Riccardo Ghiloni, *Subfield-algebraic geometry I: introduction*

  18:00–18:30 José F. Fernando, *Subfield-algebraic geometry II: main results*

  18:30–19:00 Enrico Savi, *Subfield-algebraic geometry III: the $\mathbb{Q}$-algebraicity problem*

- Tuesday – September 5th

  14:30–15:00 Antonio Carbone, *Desingularization of semialgebraic sets*

  15:00–15:30 Luis José Santana Sánchez, *Duality of divisors and curves on Mori dream spaces*

  15:30–16:00 Juliusz Banecki, *Extensions of k-regulous functions from two-dimensional varieties*

  coffee break

  16:30–17:00 Wojciech Kucharz, *Approximation and Homotopy in Regulous Geometry*

  17:00–17:30 Stanisław Spodzieja, *Effective Bertini theorem and formulas for multiplicity and the local Łojasiewicz exponent*

  17:30–18:00 Krzysztof Nowak, *Solution to a problem of pulling back singularities*

  18:00–18:30 Tadeusz Krasiński, *Jump of the Milnor number in linear deformations and δ-constant families of curve singularities*

- Thursday – September 7th

  14:00–14:30 Marcin Bilski, *Hartogs-type theorems in real algebraic geometry*

  14:30–15:00 Evelia Rosa García Barroso, *Combinatorial study of Morsifications of real univariate singularities*

  15:00–15:30 Janusz Gwoździewicz, *On some regularity condition*

  15:30–16:00 Tomasz Kowalczyk, *On sums of even powers of polynomials*

  coffee break

  16:30–17:00 Krzysztof Kurdyka, *A Bochnak-Siciak theorem for Nash functions over real closed fields*

  17:00–17:30 Maria-Angeles Zurro, *Waring decomposition of real binary forms and Brion's formula*

  17:30–18:00 Wiesław Pawłucki, *Strict C^p-triangulation of sets definable in o-minimal structures*

  18:00–18:30 Andrzej Lenarcik, *Remarks on the Łojasiewicz exponent and polar invariants of real curve*

Extensions of k-regulous functions from two-dimensional varieties

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Abstract

Even though Regulous Geometry has already undergone massive development since its invention, it still admits plenty of unsolved fundamental problems. In particular it is unknown whether every k-regulous function on a closed subvariety arises as a restriction of a k-regulous function defined on the entire ambient variety. We discuss cases in which the problem has been settled, in particular we focus on a recent result solving it completely in dimension 2.

[1] Banecki J., *Extensions of k-regulous functions from two-dimensional varieties*, arXiv:2303.02481 (2023).

[2] Fichou G., Monnier J.P., Quarez R., *Continuous functions on the plane regular after one blowing-up*, Mathematische Zeitschrift 285 (2017), no. 1, 287–323.

[3] Kollár J., Nowak K., *Continuous rational functions on real and p-adic varieties*, Mathematische Zeitschrift 279 (2015), no. 1, 85–97.

Spectral spaces in o-minimal structures

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joint work with J. F. Fernando and D. Palacin

Abstract

Normal spectral spaces form a topological context related with several areas of mathematics, specially with Real algebraic geometry. In the late 1970s, the real spectra of a ring was introduced by M. Coste and M.F. Roy as a substitute in real algebraic geometry for the Zariski spectra of a ring in ordinary algebraic geometry. The real spectra associated to a semialgebraic set X is an object whose *standard* topological properties yield information about *semialgebraic* topological properties of X.

Spectral spaces within Model Theory were first considered by A. Pillay, who introduced the so-called *o-minimal spectra* of definable sets, mainly as a tool to develop sheaf theories in the o-minimal setting.

The purpose of this talk is to relate topological properties of the o-minimal spectrum (a normal spectral space) with some relevant model-theoretic notions. Moreover, if time permits we will see that even in abstract model theoretic situations beyond o-minimality it is possible to associate meaningfully a normal spectral space.

Combinatorial study of Morsifications of real univariate singularities

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joint work with A. Bodin, P. Popescu-Pampu and M.S. Sorea

Abstract

We study a broad class of morsifications of germs of univariate real analytic functions. We characterize the combinatorial types of the resulting Morse functions by valuative methods, via planar contact trees constructed from Newton-Puiseux roots of the polar curve of the morsification.

Hartogs-type theorems in real algebraic geometry

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joint work with J. Bochnak and W. Kucharz

Abstract

Let $f : X \to \mathbb{R}$ be a function defined on a connected nonsingular real algebraic set X in $\mathbb{R}^n$. It turns out that regularity of f can be detected by controlling the restrictions of f to algebraic curves or surfaces in X. If $\dim X \geqslant 2$, then f is a regular function whenever $f|_C$ is a regular function for every algebraic curve C in X that is homeomorphic to the unit circle and has at most one singularity. If $\dim X \geqslant 3$, then f is a regular function whenever $f|_S$ is a regular function for every nonsingular algebraic surface S in X that is homeomorphic to the unit 2-sphere.

Global multiplicity, special closure and non-degeneracy of gradient maps

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joint work with J.A.C. Huarcaya
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Abstract

Given a polynomial map $F : \mathbb{C}^n \to \mathbb{C}^p$ with finite zero set, $p \geqslant n$, we introduce the notion of global multiplicity associated to F, which is analogous to the multiplicity of ideals in Noetherian local rings. This notion allows to characterize numerically the Newton non-degeneracy at infinity of F. This fact motivates us to study a combinatorial inequality concerning the normalized volume of global Newton polyhedra and to characterize the corresponding equality using special closures. We also study the Newton non-degeneracy at infinity of gradient maps and discuss some implications of our study with the index of real polynomial vector fields and the estimation of Łojasiewicz exponents at infinity.

Desingularization of semialgebraic sets

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joint work with J.F. Fernando

Abstract

Hironaka's resolution of singularities of algebraic varieties (over a field of characteristic 0) is a widespread celebrated discipline that has many applications in many areas of Mathematics. When the ground field is $\mathbb{R}$ the general approach consists of the following: *Given a real algebraic set $X \subset \mathbb{R}^n$, one finds a non-singular real algebraic set $X' \subset \mathbb{R}^m$ together with a proper polynomial map $f : X' \to X$ that is a biregular isomorphism outside the set of singular points of X.*

If $\mathcal{S} \subset \mathbb{R}^n$ is a semialgebraic set, what does it mean to desingularize $\mathcal{S}$? Unlike the algebraic case, a semialgebraic set $\mathcal{S}$ may have a boundary (that is still a semialgebraic set) and the boundary may have multiple irreducible components. Thus, we must be careful with the choice of the class of 'non-singular semialgebraic domains' to represent semialgebraic sets as the image under proper semialgebraic maps 'as much regular as possible'. If $\mathcal{S}$ is closed we can choose as 'non-singular semialgebraic domains' Nash manifolds with corners and as maps proper polynomial maps. If the involved semialgebraic set $\mathcal{S}$ is not locally closed we have to change the 'non-singular semialgebraic domains' (using Nash quasi-manifolds with corners instead of Nash manifolds with corners) and the involved proper maps are in general only Nash maps instead of polynomial maps.

We also construct the Nash double $D(\mathcal{Q})$ of a Nash manifold with corners $\mathcal{Q} \subset \mathbb{R}^m$, which generalizes the Nash double of a Nash manifold with boundary. This construction and the previous result allow, when $\mathcal{S}$ is closed, to obtain a branched Nash covering $f : D(\mathcal{Q}) \to \mathcal{S}$ whose fibers have constant cardinality outside of the ramification locus.

If time permits, we will show several applications of our results:

- Nash approximation of continuous semialgebraic maps whose target spaces are Nash manifolds with corners.
- Representation of compact semialgebraic sets connected by analytic paths as images under Nash maps of closed unit balls.

- Weak desingularization of closed semialgebraic sets using Nash manifolds with (smooth) boundary.
- Explicit construction of Nash models for compact orientable smooth surfaces.

Subfield-algebraic geometry II: main results

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Abstract

Let L be either an algebraically closed field of characteristic zero or a real closed field, and let K be any subfield of L. Let $\overline{K}^{\blacksquare}$ be the algebraic closure of K in the first case and the real closure of K in the second case. We identify $\overline{K}^{\blacksquare}$ with the algebraic closure of K in L. Given an algebraic set $X \subset L^n$ that can be described using finitely many polynomials of $\overline{K}^{\blacksquare}[\mathtt{x}_1, \ldots, \mathtt{x}_n]$ we explain a procedure to compute geometrically the smallest K-algebraic subset of L^n that contains X, that is, the smallest algebraic subset of L^n that contains X and can be described using finitely polynomials of $K[\mathtt{x}_1, \ldots, \mathtt{x}_n]$. In 1974 Stengle approached a similar problem (from the algebraic point of view) when L is a real closed field and K is endowed with the unique ordering induced by L in K. He provided a real Nullstellensatz to compute the zero ideal (in $L[\mathtt{x}_1, \ldots, \mathtt{x}_n]$) for algebraic sets $X \subset L^n$ that can be described using polynomials with coefficients in K. We will pay special attention to those algebraic subsets of L^n whose zero ideal in $L[\mathtt{x}_1, \ldots, \mathtt{x}_n]$ can be generated by finitely many polynomials with coefficients in K and we will show that this class is very restrictive (due to the strong properties they have). If time allows it we will state some properties concerning $L|K$-regularity and $L|K$-Jacobian criterion of K-algebraic sets.

Subfield-algebraic geometry I: introduction

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Abstract

Let L be either an algebraically closed field of characteristic zero or a real closed field, and let K be any subfield of L. Given a subset X of L^n, we say that $X \subset L^n$ is a K-algebraic set if it is the zero set in L^n of a family of polynomials in the subring $K[\mathtt{x}] = K[\mathtt{x}_1, \ldots, \mathtt{x}_n]$ of $L[\mathtt{x}]$. We are interested in studying the algebraic geometry of K-algebraic sets $X \subset L^n$ using only polynomials in $K[\mathtt{x}]$, and comparing it with the usual algebraic geometry of $X \subset L^n$ in which polynomials of the entire ring $L[\mathtt{x}]$ are used. This study generates a new algebraic geometry, we call subfield-algebraic geometry, which is particularly rich and interesting in the case L is a real closed field and K is not a real closed subfield of L, such as $K = \mathbb{Q}$. We present here some basic notions, results and examples, such as the K-Zariski topology, the K-irreducible components and the K-dimension, and compare them with the usual ones.

On some regularity condition

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joint work with B. Gryszka

Abstract

Let $\mathbb{K}$ be an uncountable field of characteristic zero and let f be a function from $\mathbb{K}^n$ to $\mathbb{K}$. We show that if the restriction of f to every affine plane $L \subset \mathbb{K}^n$ is regular, then f is a regular function.

On sums of even powers of polynomials

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joint work with Julian Vill

Abstract

Let n be a positive integer. For a (commutative) ring A we define its n-th Pythagoras number $p_n(A)$ as the smallest positive integer g such that any sum of n-th powers in A can be expressed as a sum of at most g n-th powers in A. If such number does not exist we put $p_n(A) = \infty$.

During the talk I will discuss main differences between quadratic forms and forms of higher degree. Then, I will show that

$$p_{2d}(\mathbb{R}[x_1, \ldots, x_n]) = +\infty$$

provided that $n \geqslant 2$ and $d \geqslant 1$. This partially solves Problem 8 from [1]. I will also discuss the case of a single variable as well as the ring of formal power series in 2 variables.

[1] Choi M-D., Dai Z.D., Lam T.Y., Reznick B., *The Pythagoras number of some affine algebras and local algebras*, Journal für die reine und angewandte Mathematik 336 (1982), 45–82.

Jump of the Milnor number in linear deformations and δ-constant families of curve singularities

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joint result with A. Zakrzewska

Abstract

The jump of the Milnor number of an isolated singularity f_0 is the minimal non-zero difference between the Milnor numbers of f_0 and a generic element of its deformations. We characterize plane curve singularities for which this jump is equal to one in the class of linear deformations. We use this result to determine topological types of singularities in δ-constant linear deformations in some classes of singularities.

Approximation and Homotopy in Regulous Geometry

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Abstract

Let X, Y be nonsingular real algebraic sets. A map $\varphi\colon X \to Y$ is said to be k-regulous, where k is a nonnegative integer, if it is of class $\mathcal{C}^k$ and the restriction of φ to some Zariski open dense subset of X is a regular map. Assuming that Y is uniformly rational, and $k \geqslant 1$, we prove that a $\mathcal{C}^\infty$ map $f\colon X \to Y$ can be approximated by k-regulous maps in the $\mathcal{C}^k$ topology if and only if f is homotopic to a k-regulous map. The class of uniformly rational real algebraic varieties includes spheres, Grassmannians and rational nonsingular surfaces, and is stable under blowing up nonsingular centers. Furthermore, taking $Y = \mathbb{S}^p$ (the unit p-dimensional sphere), we obtain several new results on approximation of $\mathcal{C}^\infty$ maps from X into $\mathbb{S}^p$ by k-regulous maps in the $\mathcal{C}^k$ topology, for $k \geqslant 0$.

A Bochnak-Siciak theorem for Nash functions over real closed fields

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joint work with W. Kucharz

Abstract

Let R be a real closed field. We prove that if R is uncountable, then a function $f\colon U \to R$ defined on an open semialgebraic set U in R^n, with $n \geqslant 2$, is a Nash function whenever for every affine 2-plane Q in R^n the restriction $f|_{U\cap Q}$ is a Nash function (some condition on the shape of U is required if R is not Archimedean). This is an analog of the Bochnak–Siciak theorem established in the real analytic setting. We also provide an example showing that uncountability of R is essential.

Remarks on the Łojasiewicz exponent and polar invariants of real curve

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Abstract

Let $f \in \mathbb{R}\{X, Y\}$ be a convergent series without constant term such that the analytic function $f : (\mathbb{R}^2, 0) \to (\mathbb{R}, 0)$ defined by him has isolated zero. Let us consider a generic polar curve $\partial f = a(\partial f/\partial X) + b(\partial f/\partial Y) = 0$ $(a, b \in \mathbb{R})$. To every real branch of ∂f with parametrization $\gamma : (\mathbb{R}, 0) \to (\mathbb{R}^2, 0)$ we assign the *real polar invariant*

$$\frac{\operatorname{ord}(f \circ \gamma)}{\operatorname{ord} \gamma} .$$

By $Q^{\mathbb{R}}(f)$ we denote the set of all real polar invariants of f.
Gwoździewicz [1], [2] showed that $Q^{\mathbb{R}}(f) \neq \varnothing$. We define the *Łojasiewicz exponent* of f as

$$\begin{aligned} l_0(f) \quad = \quad & \inf\{\lambda > 0 : |f(x, y)| \geqslant C \max\{|x|, |y|\}^{\lambda} \\ & \text{near } \ 0 \in \mathbb{R}^2 \quad \text{for } \ C > 0\}. \end{aligned}$$

Gwoździewicz proved that $l_0(f) = \max Q^{\mathbb{R}}(f)$. We can treat the series $f(X, Y)$ as an element of the ring $\mathbb{C}\{X, Y\}$. Then we can consider the analogous set $Q^{\mathbb{C}}(f)$ of *complex polar invariants* [3]. Obviously $Q^{\mathbb{R}}(f) \subset Q^{\mathbb{C}}(f)$. Moreover, $\max Q^{\mathbb{C}}(f) < \infty$ if and only if f has no multiple factors in $\mathbb{C}\{X, Y\}$.

Results describing relations between $Q^{\mathbb{R}}(f)$ and $Q^{\mathbb{C}}(f)$ will be presented. The techniques as Newton diagrams and characteristics of branches will be applied.

[1] Gwoździewicz J., *Wykładnik Łojasiewicza funkcji analitycznej o zerze izolowanym*, Uniwersytet Jagielloński, PhD thesis, 1995.

[2] Gwoździewicz J., *The Łojasiewicz eksponent of an analytic function at an isolated zero*, Commentarii Mathematici Helvetici 74 (1999), 364–375.

[3] Teissier B., *Variétés polaires*, Inventiones Mathematicae 40 (1977), 267–292.

Solution to a problem of pulling back singularities

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Abstract

The aim of my talk is to give an affirmative general solution to the following pullback problem. Consider a finite holomorphic map germ $\varphi : (\mathbb{C}^n, 0) \to (\mathbb{C}^n, 0)$ and an analytic subvariety germ X in the target. Then if the preimage $Y = \varphi^{-1}(X)$, taken with the reduced structure, is smooth, so is X. The case, where Y is not contained in the ramification divisor Z of φ, was established by Ebenfelt–Rothschild (2007) and afterwards by Lebl (2008) and Denkowski (2016). The hypersurface case was achieved by Giraldo–Roeder (2020) and recently by Jelonek (2023).

Strict C^p-triangulation of sets definable in o-minimal structures

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Abstract

Let R be any real closed field expanded by some o-minimal structure. Let $f : A \to R^d$ be a definable and continuous mapping defined on a definable, closed, bounded subset A of R^n. Let p be any positive integer. The main result tells that then there exists a finite simplicial complex $\mathcal{T}$ in R^n and a definable homeomorphism $h : |\mathcal{T}| \to A$, where $|\mathcal{T}| := \bigcup \mathcal{T}$, such that for each simplex $\Delta \in \mathcal{T}$, the restriction of h to its relative interior $\mathring{\Delta}$ is a C^p-embedding of $\mathring{\Delta}$ into R^n and moreover both h and $f \circ h$ are of class C^p in the sense that they have definable C^p-extensions defined on an open definable neighborhood of $|\mathcal{T}|$ in R^n. We call a pair $(\mathcal{T}, h)$ *a strict C^p-triangulation of A.*

Duality of divisors and curves on Mori dream spaces

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Abstract

Let X be a normal $\mathbb{Q}$-factorial variety with zero irregularity. For any integer $0 \leqslant k \leqslant n-1$, we consider the cone D_k of numerical classes of effective divisors on X whose stable base locus has codimension larger than k and we set $\mathcal{D}_k$ to be its closure in $\mathrm{N}^1(X)_{\mathbb{R}}$. We obtain the filtration

$$\mathrm{N}ef(X) = \mathcal{D}_{n-1} \subseteq \cdots \subseteq \mathcal{D}_1 \subseteq \mathcal{D}_0 = \overline{\mathrm{Eff}(X)}.$$

We consider an analogous filtration of cones of pseudo-effective curves:

$$\overline{\mathrm{N}E(X)} = \mathcal{C}_{n-1} \supseteq \cdots \supseteq \mathcal{C}_1 \supseteq \mathcal{C}_0,$$

where $\mathcal{C}_k$ is the closure of the cone in $N_1(X)_{\mathbb{R}}$ generated by classes of *k-moving* curves, that is, classes of irreducible curves moving in a family that sweeps out an $(n-k)$-dimensional subvariety of X. It is known that $\mathcal{C}_{n-1}$ and $\mathcal{C}_0$ are respectively dual to $\mathcal{D}_{n-1}$ and $\mathcal{D}_0$ under the standard intersection product. In general, duality for intermediate cones is not expected. However, [2] proved that for toric varieties, we obtain duality if we also consider curves sweeping out varieties in some small modification of X. We call this *weak duality*. Later, [1] proved that this weak duality holds for Mori dream spaces and asked if, as the pseudo-effective cone, the $\mathcal{D}_k$ cones are also polyhedral.

In this talk we give a positive answer to Choi's question, by relating the filtration of $\overline{\mathrm{Eff}(X)}$ to the Mori chamber decomposition of X. We also present examples where a *strong duality* holds.

[1] Choi S.R., *Duality of the cones of divisors and curves*, Mathematical Research Letters 19 (2012), no. 2, 403–416.

[2] Payne S., *Stable base loci, movable curves, and small modifications, for toric varieties*, Mathematische Zeitschrift 253 (2006), no. 2, 421–431.

Subfield-algebraic geometry III: the $\mathbb{Q}$-algebraicity problem

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Abstract

In 2020, Parusiński and Rond proved that every algebraic set $V \subset \mathbb{R}^n$ is homeomorphic to a $\overline{\mathbb{Q}}^r$-algebraic set $V' \subset \mathbb{R}^n$, where $\overline{\mathbb{Q}}^r$ denotes the field of real algebraic numbers. The aim of this talk is to provide some classes of algebraic sets that positively answer the following open problem:

$\mathbb{Q}$-ALGEBRAICITY PROBLEM: (Parusiński, 2022) Is every algebraic set $V \subset \mathbb{R}^n$ homeomorphic to some $\mathbb{Q}$-algebraic set $V' \subset \mathbb{R}^m$, with $m \geqslant n$?

We introduce the notion of $\mathbb{Q}$-determined $\mathbb{Q}$-algebraic sets. Roughly speaking, a $\mathbb{Q}$-algebraic set is $\mathbb{Q}$-determined if its local behaviour at nonsingular points is described by polynomial equations with rational coefficients via the $\mathbb{R}|\mathbb{Q}$-Jacobian criterion of $\mathbb{Q}$-algebraic sets. This notion is crucial to develop $\mathbb{Q}$-algebraic approximation techniques and to provide a version over $\mathbb{Q}$ of the relative Nash-Tognoli theorem. Latter result, combined with a version over $\mathbb{Q}$ of the classical blowing down lemma, allows us to give a complete positive answer to the above $\mathbb{Q}$-ALGEBRAICITY PROBLEM in the case of nonsingular algebraic sets and algebraic sets with isolated singularities.

Effective Bertini theorem and formulas for multiplicity and the local Łojasiewicz exponent

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Abstract

The classical Bertini theorem on generic intersection of an algebraic set with hyperplanes states the following:

Let X be a nonsingular closed subvariety of $\mathbb{P}^n_k$, where k is an algebraically closed field. Then there exists a hyperplane $H \subset \mathbb{P}^n_k$ not containing X and such that the scheme $H \cap X$ is regular at every point. Furthermore, the set of hyperplanes with this property forms an open dense subset of the complete linear system $|H|$ considered as a projective space.

We show that one can effectively indicate a finite family of hyperplanes H such that at least one of them satisfies the assertion of the Bertini theorem, provided the characteristic of the field k is equal to zero. As an application of the method used in the proof we give effective formulas for the multiplicity and the Łojasiewicz exponent of polynomial mappings.

Waring decomposition of real binary forms and Brion's formula

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joint work with M. Ansola and A. Diaz-Cano

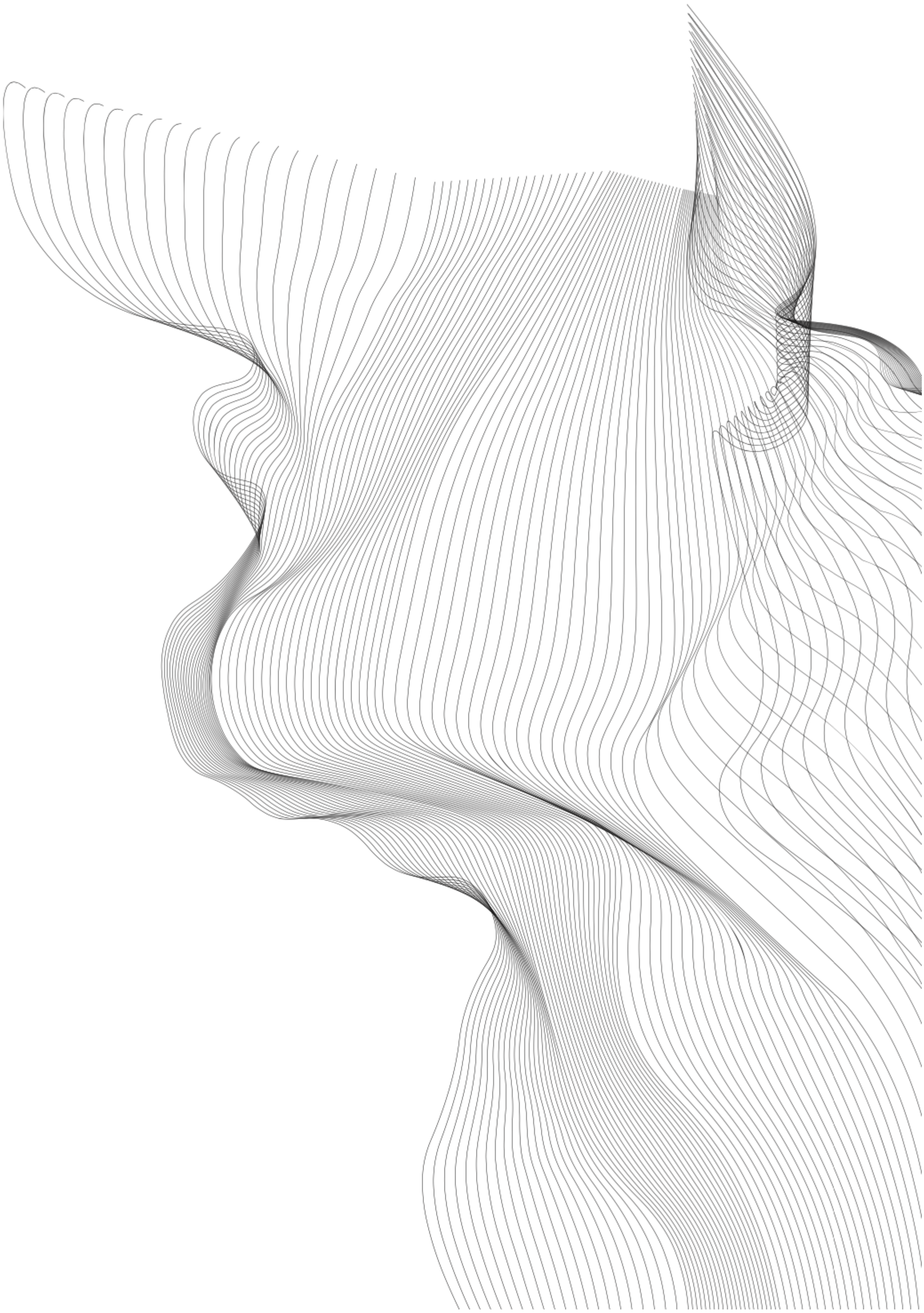
Abstract

The Waring Problem over polynomial rings studies the decomposition of a homogeneous polynomial $p \in K[x_1, \ldots, x_n]$ of degree d in the variables $x_1, \ldots, x_n$ as a linear combination of d-th powers of linear forms with coefficients in the field K. The case of real or complex coefficients is especially important in applications.

In the talk we will focus on the case $n = 2$ where there is a very efficient algorithm to calculate such decompositions, see [1]. We will show how the use of such decompositions allows to obtain a parametric integration formula over triangles in semi-algebraic neighborhoods of p for a certain topology in the space of real binary forms of degree d. This formula is the extension of the formulas given in [2], where the parametric behavior of the integrals was not studied.

[1] Ansola M., DĂaz-Cano A., Zurro M.A., *Semialgebraic sets and real binary forms decompositions*, Journal of Symbolic Computation 107 (2021), 209–220.

[2] Baldoni V., Berline N., De Loera J.A., Köppe M., Vergne M., *How to integrate a polynomial over a simplex*, Mathematics of Computation 80 (2011), 297–325.



Thematic section

M

Miscellanea

ORGANIZERS:

Andrzej Komisarski (Uniwersytet Łódzki)

Kamil Niedziałomski (Uniwersytet Łódzki)

SCHEDULE OF THE SECTION
Miscellanea

- Thursday – September 7th

  14:30–15:00 Piotr Felisiak, *Generalized multiset theory and its applications*

  15:00–15:30 Piotr Mizerka, *Induction of spectral gaps for the cohomological Laplacians of* $\mathrm{SL}_n(\mathbb{Z})$ *and* $\mathrm{SAut}(F_n)$

  15:30–16:00 Leonard Mushunje, *High Dimensional Functional Data Analysis via Algebraic Geometry*

  coffee break

  16:30–17:00 Bożena Piątek, *Some types of generalized nonexpansive mappings and normal structure*

  17:00–17:30 Piotr Puchała, *A few remarks about young measures associated with bounded Borel functions*

  17:30–18:00 Adam Paszkiewicz, *The convex Peano curve does exist!*

Generalized multiset theory and its applications

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Abstract

The talk presents a kind of generalized multisets, existing by an axiomatic theory firstly presented by [4], and applications of these multisets in linear algebra, such as providing equivalents of classical vector operations and Gaussian elimination. Further, these generalized multisets will be called *fogs*.

A multiset is a collection of objects, in which these objects may occur more than once, *i.e.*, the collection may include several indistinguishable copies of an object. Objects contained by a multiset are called *elements* of this multiset. The number of times an element occurs in a multiset is called the *multiplicity* of the element and is a natural number. A Zermelo–Fraenkel set may be seen as a special case of a multiset, such that every element of such a multiset is with the multiplicity equal to 1. The *cardinality* of a multiset is the sum of the multiplicities of its elements. An excellent survey of literature on the multiset theories is given by W.D. Blizard in [3]. A popular introduction to the concept of multisets is [6].

Several generalizations of multisets were developed. Typically, they are based on an extension of the range of multiplicities. This talk will consider formal, axiomatic generalizations of multisets. Such generalized multisets, existing by axiomatic theories, will be further called *α-multisets*. There are not so many theories of α-multisets: in Blizard's work, [1], the multiplicity of an element is allowed to be a positive real number; in a formal theory given by him in [2], the value of a multiplicity can be positive or negative integer, what implies a possibility of negative membership; finally, Felisiak, Qin and Li in [4] propose a theory of multisets where the multiplicities are allowed to be arbitrary real numbers, that is, including negative ones, and this theory describes the fogs.

The talk explains why the fog theory may be preferred over the classical definition of multisets and their generalizations.

With fogs, it is possible to perform an equivalent of Gaussian elimination on a linear equations system (LES). Let us further call this process *foggy elimination*. An attractive feature of the foggy elimination is that

the eliminated variables (in matrix notation, zero entries) simply are not contained by the elements of the LES representation, thus these elements gradually simplify during the elimination process, while in the case of matrix notation, the matrix size remains the same for all steps of Gaussian elimination. Similarly, foggy elimination seems particularly well suited for sparse LES, since all zero coefficients in LES does not need to be represented in foggy elimination at all. Another attractive feature is that, since the writing order of elements of foggy LES representation does not matter, we do not need to carry out something analogous to the row swap operation in Gaussian elimination. One of the biggest advantages of foggy elimination is that the terms of equations do not need to be sorted by the lower indices, since as usual in fogs, the writing order of elements does not matter. This is in contrast to the Gaussian elimination, where the lower indices must correspond to column indices in the augmented matrix.

An algorithm for foggy elimination is developed, where partial pivoting is applied. The algorithm is implemented in the Python programming language.

[1] Blizard W.D., *Real-valued multisets and fuzzy sets*, Fuzzy Sets and Systems 33 (1989), no. 1, 77–97.

[2] Blizard W.D., *Negative membership*, Notre Dame Journal of Formal Logic 31 (1990), no. 3, 346–368.

[3] Blizard W.D., *The development of multiset theory*, Modern Logic 1 (1991), no. 4, 319–352.

[4] Felisiak P.A., Qin K., Li G., *Generalized multiset theory*, Fuzzy Sets and Systems 380 (2019), 104–130.

[5] Goguen J.A., *L-fuzzy sets*, Journal of Mathematical Analysis and Applications 18 (1967), no. 1, 145–174.

[6] Knuth D.E., *The art of computer programming*, 3rd ed., vol. 2: Seminumerical Algorithms, Addison Wesley 1998.

Induction of spectral gaps for the cohomological Laplacians of $\mathrm{SL}_n(\mathbb{Z})$ and $\mathrm{SAut}(F_n)$

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joint work with Marek Kaluba
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Abstract

In 2020 Bader and Nowak [1] showed a sufficient condition for vanishing of group cohomology with unitary coefficients. The condition involves the existence of a positive spectral gap for the first cohomological Laplacian, defined in a group ring setting. In degree one this method provides an alternative to Ozawa's [4] way to prove Kazhdan's property (T).

In this talk we focus on $\mathrm{SL}_n(\mathbb{Z})$ and $\mathrm{SAut}(F_n)$, the special linear group of $n \times n$ matrices over integers and the special automorphism group of the free group on n generators. We present a method to induce spectral gaps for $G_n \in \{\mathrm{SL}_n(\mathbb{Z}), \mathrm{SAut}(F_n)\}$ for cohomological Laplacians in degree one. We were inspired by the work of Kaluba, Kielak, and Nowak [2] who proved property (T) for G_n using the induction method for the Laplacian introduced by Ozawa. Using the presentations of G_n with generators being the elementary matrices for $\mathrm{SL}_n(\mathbb{Z})$ and Nielsen transvections for $\mathrm{SAut}(F_n)$, we decompose the Laplacians in degree one into summands which behave well after applying the symmetrization technique to them. This allows us to obtain a lower bound for the desired spectral gap for G_n, once we know that such a gap exists for a specific summand of the Laplacian for G_m, whenever $n \geqslant m$.

As an application of the induction technique, we were able to find explicit lower bounds for the spectral gaps for $G_n = \mathrm{SL}_n(\mathbb{Z})$ in all possible cases, that is $n \geqslant 3$. Our approach was motivated by the existence of a positive spectral gap for the Laplacian in degree one for $\mathrm{SL}_3(\mathbb{Z})$ [3] which we showed last year. Recently, we were able to find a bound for the spectral gap of the suitable part of this Laplacian which allowed us to use the induction technique and obtain the bounds for all $n \geqslant 3$. This constitutes in particular an alternative proof of property (T) for $\mathrm{SL}_n(\mathbb{Z})$.

[1] Bader U., Nowak P.W., *Group algebra criteria for vanishing of cohomology*, Journal of Functional Analysis 279 (2020), no. 11.

[2] Kaluba M., Kielak D., Nowak P.W., *On property (T) for* $\mathrm{Aut}(F_n)$ *and* $\mathrm{SL}_n(\mathbb{Z})$, Annals of Mathematics 193 (2021), no. 2, 539–562.

[3] Kaluba M., Mizerka P., Nowak P.W., *Spectral gap for the cohomological Laplacian of* $\mathrm{SL}_3(\mathbb{Z})$, arXiv:2207.02783 (2022).

[4] Ozawa N., *Noncommutative real algebraic geometry of Kazhdan's property (T)*, Journal of the Institute of Mathematics of Jussieu 15 (2016), no. 1, 85–90.

High Dimensional Functional Data Analysis via Algebraic Geometry

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Abstract

When regressing high-dimensional functional data, challenges are often encountered mainly for the in-sample than out-sample results and even worse when subjected to the curse of dimensionality. For example, on the in-sample, minimal bounds on the eigenvalues of the covariance matrix for the covariates, when using ridge regression, are not generally considered. This study aims to explore the in-sample MSPE properties of different regression methods (except ridge regression) and understand whether the eigenvalue lower bounding conditions are generally avoidable in high-dimensional Hilbert settings.

The convex Peano curve does exist!

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Abstract

We present the following main result with a number of applications and conjectures. For any convex and compact set $\mathbb{T} \subset \mathbb{R}^2$, there exists a continuous surjection $f : [0,1] \to \mathbb{T}$, such that $f(I)$ is convex for any interval $I \subset [0,1]$. If the interior $\mathrm{int}\mathbb{T} \neq \varnothing$ then one can obtain additionally $\mathrm{int} f(I) \neq \varnothing$ for any open interval $I \in [0,1]$.

Some types of generalized nonexpansive mappings and normal structure

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Abstract

We consider relations between a normal structure of a Banach space and the fixed point property for various classes of generalized nonexpansive mappings under additional assumptions, such as that of continuity. In this way we answer some open questions about the behaviour of such maps (see [1] and [2]).

[1] Betiuk-Pilarska A., Wiśnicki A., *On the Suzuki nonexpansive-type mappings*, Annals of Functional Analysis 4 (2013), 72–86.

[2] Fetter H., Llorens-Fuster E., *Jaggi nonexpansive mappings revisited*, Journal of Nonlinear Convex Analysis 18 (2017), 1771–1779.

A few remarks about young measures associated with bounded Borel functions

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Abstract

A probabilistic characterization of Young measures associated with bounded Borel functions is presented with some possible applications. The density of a Young measure is defined and illustrated with examples. The relations between convergence of sequences of homogeneous Young measures with densities and convergence of sequences of these densities are investigated.

[1] Fryszkowski A., *Continuous selections for a class of nonconvex multivalued maps*, Studia Mathematica 75 (1983), 163–174.

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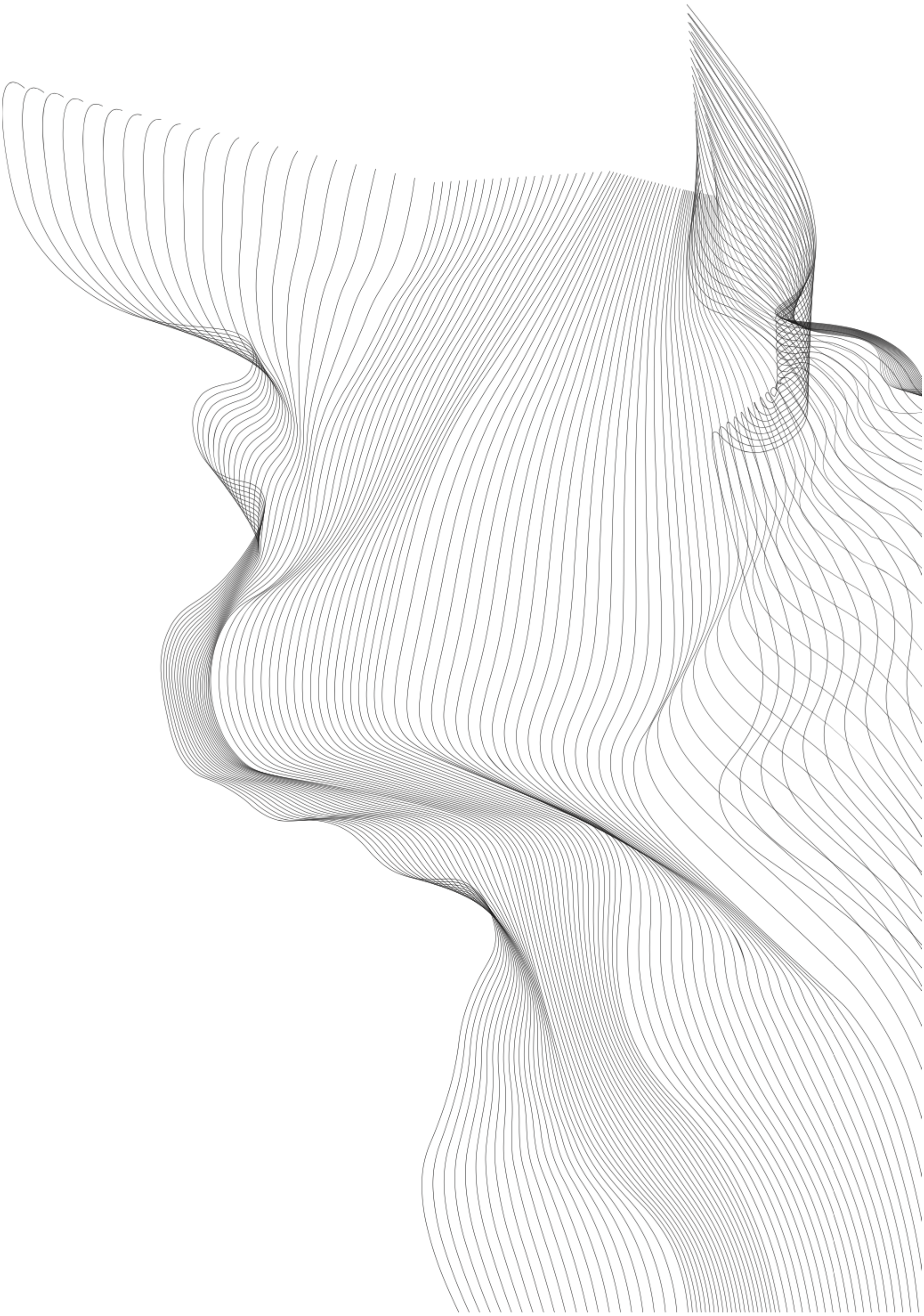
[4] Puchała P., *An elementary method of calculating an explicit form of Young measures in some special cases*, Optimization 63 (2014), 1419–1430.

[5] Puchała P., *A simple characterization of Young measures and weak L^1 convergence of their densities*, Optimization 66 (2017), 197–203.

[6] Puchała P., *Weak convergence of the sequences of Young measures with densities*, submitted.

[7] Roubíček T., *Relaxation in optimization theory and variational calculus*, Walter de Gruyter, Berlin, New York, 1997.



Discussion Panel

DM

Didactics of Mathematics

MODERATOR:

Ryszard Pawlak (Uniwersytet Łódzki)

PANELISTS:

Marianna Bosch (Universitat de Barcelona)

Piotr Błaszczyk (Uniwersytet Pedagogiczny w Krakowie)

Nuria do los Angles Climent Rodriguez (Universidad de Huelva)

Edyta Juskowiak (Uniwersytet Adama Mickiewicza w Poznaniu)

Ewa Swoboda (PWSTE Jarosław)

Tomasz Szemberg (Uniwersytet Pedagogiczny w Krakowie)

Didactics of Mathematics

- Wednesday – September 6th

  12:00–13:30

DESCRIPTION

A panel discussion is titled How to Overcome Barriers in Teaching and Learning Mathematics. In the field of mathematics education, both students and teachers encounter various barriers that need to be overcome. Therefore, it is valuable to explore the perspectives of renowned experts in mathematics didactics on this matter.

While waiting for the discussion among renowned specialists, it is possible to get a preliminary insight into how artificial intelligence (ChatGPT) "expresses itself" on this topic:

Overcoming barriers in teaching and learning mathematics requires a multifaceted approach. Firstly, fostering a positive and supportive learning environment is crucial. Teachers can create an inclusive classroom where students feel safe to ask questions and make mistakes without fear of judgment.

Secondly, employing diverse instructional strategies can help address individual learning styles and cater to the needs of different students. This may involve incorporating hands-on activities, visual aids, technology, and collaborative learning opportunities to engage students and enhance their understanding of mathematical concepts.

Furthermore, providing personalized feedback and targeted interventions can assist students in identifying and addressing their specific challenges. Teachers can offer one-on-one support, extra practice exercises, or additional resources to reinforce learning and bridge knowledge gaps.

Effective communication and collaboration between teachers, students, and parents also play a significant role. Regular progress updates, open lines of communication, and involving parents in their child's mathematical education can create a supportive network that encourages growth and persistence.

Lastly, promoting a growth mindset is essential. Encouraging students to embrace challenges, view mistakes as opportunities for learning, and develop a resilient attitude towards mathematics can foster a positive mindset and empower them to overcome obstacles. By implementing these strategies, educators can help students navigate and overcome barriers in teaching and learning mathematics, promoting a deeper understanding and appreciation for the subject.

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